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RULES AND EXAMPLES

IN

ALGEBRA.

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RULES AND EXAMPLES

IN

ALGEBRA.

PART I

BY

THE REV. T. DALTON, M.A.

ASSISTANT MASTER AT ETON COLLEGE.

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PREFACE.

In this book I have attempted to apply to Algebra the plan which has been found useful in my "Arithmetic":—the rules are stated briefly, so as not to be a substitute for, but to *supplement*, the work of the teacher; specimen examples are given, explanations being added where they seem necessary; and numerous examples for the student's own work are arranged, it is hoped, in such a manner as to afford him all the practice he will need.

The indication of any difficulties or errors will be very thankfully received.

T. D.

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ALGEBRA.

CHAPTER I.

Numerical Substitution.

In Algebra we work with *figures and letters*, just as in Arithmetic we work with figures only.

The *symbols* used in Arithmetic are used in Algebra with the same meaning, but in Algebra there are also others which are not used in Arithmetic.

The multiplication of letters may be denoted like that of figures by the sign $\times$, but it may also be expressed in two other ways:—(1) by placing a point between the letters; thus, $a.b$ denotes $a \times b$; $a.b.c$ denotes $a \times b \times c$: (2) by writing the letters together without any sign between them; thus abc means the same as $a \times b \times c$, or $a.b.c$. The first of these two methods cannot be employed in Arithmetic, because the point might be mistaken for a decimal point; thus if we were to write 8×9 in the form 8·9, it would be taken for $8\frac{9}{10}$. The second of the two methods is also inapplicable to Arithmetic, because two or more figures without a sign between them denote a number; thus 8×9 cannot be written 89, for this would mean eighty-nine. Thus, then, while there is only one sign in Arithmetic by which we may denote the product of two or more numbers, we have in Algebra three methods; —the continued product of a, b, and c may be written

$a \times b \times c$, or $a.b.c$, or abc, the last of which is that generally adopted.

In the first exercise the student will find that some fixed numerical value is given to the various letters employed; this is for the sake of rendering the first treatment of Algebraical expressions more easy to him: in the chapters and exercises which follow, he will find that the letters are perfectly general, and have no numerical values attached to them.

When the numerical value of an Algebraical expression is to be found, you substitute for each letter its given value, and then simplify the expression as in Arithmetic.

Exs. If $a=6$, $b=5$, $c=4$, $d=3$, $e=2$, $f=1$, $g=0$, find the numerical value of

(1) $a+b-c-d+e-f-g$,

(2) $acd+4bde+3bcf+4ceg$,

and (3) $\frac{a+c}{b}-\frac{d+f}{c}+\frac{ae-3cg}{2df}+\frac{bd-ef}{de}$.

(1) $a+b-c-d+e-f-g$
$=6+5-4-3+2-1-0$
$=13-8=5$. Ans.

(2) $acd+4bde+3bcf+4ceg$
$=72+120+60+0$
$=252$. Ans.

It will be observed that the value of $4bde$ is found by multiplying together 5, 3, and 2; and then multiplying the result by 4: also, that in finding the product of c, e, and g, or 4, 2, and 0, the result is 0, because the introduction of the factor 0 destroys the product of the factors which are with it.

(3) $\dfrac{a+c}{b}-\dfrac{d+f}{c}+\dfrac{ae-3cg}{2df}+\dfrac{bd-ef}{de}$

$=\dfrac{6+4}{5}-\dfrac{3+1}{4}+\dfrac{12-0}{6}+\dfrac{15-2}{6}$

$=2-1+2+2\frac{1}{6}$

$=5\frac{1}{6}$. Ans.

When a letter is multiplied by itself, it is written with a small 2 above it, called an *index;*

thus $a\times a$ is written a^2,

$x\times x$.....x^2;

similarly $a\times a\times a$ .....a^3,

$x\times x\times x\times x$.....$x^4$;

and generally, the number of the index shews how many times the letter is multiplied into itself.

Hence if $a=6$, $a^2=6\times 6=36$.

$b=4$, $b^3=4\times 4\times 4=64$.

$x=1$, $x^4=1\times 1\times 1\times 1=1$.

When various letters with indices are multiplied together, care must be taken to restrict each index to its own letter: thus, in finding the value of ab^3c^2 when $a=5$, $b=4$, $c=1$, we have $a=5$, $b^3=64$, $c^2=1$; hence $ab^3c^2=320$.

Exs. If $a=0$, $b=1$, $c=2$, $x=3$, $y=4$, $z=5$, find the numerical value of

(1) $a^2+b^2+c^2-x^2-y^2+z^2$,

(2) $a^3+3a^2b+2b^2c+7ax^2+cxy+3byz$,

and (3) $\dfrac{b^2+cx}{x+y}-\dfrac{a^3+bc^2}{a+y}+\dfrac{x^2y+b^2z-bcz-b^3}{3b+4c+3x}$.

(1) $a^3+b^2+c^3-x^3-y^2+z^2$
$=0+1+4-9-16+25$
$=5$. Ans.

(2) $a^3+3a^2b+2b^2c+7ax^2+cxy+3byz$
$=0+0+4+0+24+60$
$=88$. Ans.

(3) $\dfrac{b^3+cx}{x+y}-\dfrac{a^3+bc^2}{a+y}+\dfrac{x^2y+b^2z-bcz-b^3}{3b+4c+3x}$

$=\dfrac{1+6}{3+4}-\dfrac{0+4}{0+4}+\dfrac{36+5-10-1}{20}$

$=1-1+1\frac{1}{2}=1\frac{1}{2}$. Ans.

Exercise I.

If $a=5$, $b=4$, $c=3$, $d=2$, $e=1$, $f=0$, find the numerical value of

1. $a+b+c-d-e+f$.
2. $a-b-c+d+e-f$.
3. $2a+3b+4c+d+3e$.
4. $5a+6b-2c-4d+e-2f$.
5. $6a+b-11c-12d+3e+4f$.
6. $9b-6d+4c-a+10f$.
7. $2b+3a-2f-9e-4c$.
8. $a+b-10c-2d+4e-3f$.
9. $ab+bc+cd+de+ad$.
10. $bd-ce-be+ac-bf+ef$.
11. $ae+bf-de-ca-eb-fc$.
12. $af+be+cd-bd-ae-fd$.
13. $2ab+3bc-4cd-5de+4ef$.

14. $3ac-4ad-5af+4bc+6bd-ce.$

15. $9bf-7ce+4cd-11ae+2bc-10eb.$

16. $5bd-3ac+4be-cd-2bf+de.$

17. $4abc+5bcd-6cde-7abe.$

18. $9bce-2abd+abe-2cde-def.$

19. $3ade+4bce-adf-3bde-7cef.$

20. $2ace-3bdf-4aef+6bcd-9cde.$

If $a=4$, $b=2$, $c=0$, $x=5$, $y=3$, $z=1$, find the value of

21. $4a+6bx+cy+2xz+3by.$

22. $2cx-bxz+2axy-3byz-4ac.$

23. $aby-2bcx-3xyz+4abc-2aby.$

24. $11ac+5bx-4a-bcx+2axz.$

25. $\dfrac{a+c}{2b}+\dfrac{b+x}{a+y}-\dfrac{y+z}{a}.$

26. $\dfrac{2a+3c}{y+z}-\dfrac{3b-z}{x}+\dfrac{4x+2c}{5b}.$

27. $\dfrac{2b+y+z}{x-z}-\dfrac{b+x-2y}{3b-x}+\dfrac{3a-x-y}{y-z}.$

28. $\dfrac{ab}{z}-\dfrac{bx}{a}+\dfrac{xz}{b}+\dfrac{ac}{x}.$

29. $\dfrac{bc+xy}{a+z}-\dfrac{bx+az}{b+x}-\dfrac{3by}{c+y}.$

30. $\dfrac{2ae+3bx-4yz}{x+y}-\dfrac{2by+3xz-ay}{c+z}+\dfrac{2bxz+3xyz}{ab+cx}.$

31. $a^2+b^2+c^2+x^2+y^2+z^2.$

32. $2a^2+3b^2-4x^2+y^2+5z^2.$

33. $4a^2-b^2+7c^2+3x^2+11y^2.$

34. $5a^2-7b^2-ac+4xy+6y^2+9z^2$.

35. $a^3+b^3+c^3+x^3+y^3+z^3$.

36. $4x^3+3a^2+4a-5b^3-7b+6x^3$.

37. $x^3-2y^3+3z^4-2ab-9b^3$.

38. $a^4-6c-2x^3+4y^3-z^5$.

39. $6a^3+b^2x+bx^3+y^3z+yz^3$.

40. $2x^3-3x^2y+3xy^2+y^3-3xyz$.

41. $a^3b-3ab^2+4b^3c-8x^2y+6xz^2$.

42. $12a^2c-6ac^2+15xy^2-x^2z-2y^3$.

43. $b^4-2b^3x+3b^2x^2-2bx^3+x^4$.

44. $2y^4-4y^3z+8y^2z^2-4yz^3+z^4$.

45. $a^3bx-ab^3x+abx^3-x^3yz+xy^3z-xyz^3$.

46. $a^7b^2c+a^5b^3x+ab^3y-bx^5y^2-bxy^5+x^3yz^3$.

47. $a^2b^3+ab^3z-b^3xz^2+acx^3-b^3xy^2+ayz^3$.

48. $2a^4x-7abxy^2+12axy^3z-2bx^3y^3+abxyz$.

49. $\dfrac{2a^2+5c^2}{b}-\dfrac{3ab-4y^2}{z}+\dfrac{7z^2+2xy}{a}$.

50. $\dfrac{3ab+4bc}{az}-\dfrac{2xy-3yz}{xz}+\dfrac{a^2-ab+b^2}{yz}$.

51. $\dfrac{2x^2+5xy-y^2}{ax}+\dfrac{3a^2-2ac+7c^2}{by}+\dfrac{y^2-4yz+5z^2}{vz}$.

52. $\dfrac{x^3+3x^2y+4xy^2-5y^3}{2x^2}-\dfrac{2a^2x-3axy}{3y^2-2z^2}+\dfrac{12z^3-6y^2z+4y^3}{4z^2}$.

53. $\dfrac{2x^3-6a^2b+4ab^2}{xyz}-\dfrac{y^4-2y^2+6}{axy}+\dfrac{x^3-5x^2+3x-10}{b^2z}$.

54. $\dfrac{b^3z^3+3bz^3+5z^4}{ax} - \dfrac{2xy^3-4x^2y^2+x^4}{yz} + \dfrac{b^4-b^2x^2+by^3}{xy}$.

55. $a^3+2x^2-c^2+4x^2$.

56. $\dfrac{3b^2-4y^2}{3by-4yz} - \dfrac{2a^2y^2-3a^3c^2}{2abyz-3abcy} + \dfrac{5x^2y-2ab^2}{5xyz-2abx}$.

CHAPTER II.

ADDITION.

WHEN algebraical expressions are to be added together, they must be arranged under each other, the letters of one line being placed directly below the same letters of the previous line; if in any of the lines some of the letters do not appear, their places in those lines must be left vacant.

In adding the columns, begin with the first on the left; add together all the positive figures of each column, and then all its negative figures; take the difference between the two results, and place before it the sign +, if the sum of the positive figures be the greater, and the sign −, if the sum of the negative figures be the greater.

The figure which is placed before a letter is called its *coefficient;* if in any case a letter has no coefficient expressed, it is understood to be the figure 1.

Ex. 1. Add together

$6a+$	$7b+$	$11c+$	$4d+$	9
$2a-$	$5b+$	$8c-$	$11d-$	7
$3a-$	$5b-$	$9c$	$+$	6
$a+$	$10b-$	$3c+$	d	
$-7a-$	$6b-$	$7c-$	$5d-$	10
$5a+$	b		$-11d-$	2

In this example, in the first column the positive terms amount to $12a$, the negative term is $7a$; hence we have $5a$: in the 2nd column the positive terms amount to $17b$, the negative to $-16b$; hence we have $+1b$ or $+b$: in the 3rd column, the positive terms amount to

19c, the negative to $-19c$, and these destroy each other: in the 4th column the positive terms amount to $5d$, the negative to $-16d$, whence we have $-11d$: in the last column the positive terms amount to 15, the negative to -17, and we therefore have -2.

Ex. 2. Add together

$$4x^4+3x^3y+7x^2y^2-5xy^3+6y^4,$$
$$3x^4-8x^3y+7xy^3-12y^4,\quad -6x^4+8x^3y^2-7xy^3,$$
$$14x^3y+2x^2y^2-xy^3-y^4,\text{ and }x^4+x^3y+2xy^3+7y^4.$$

$$\begin{array}{r}
4x^4+3x^3y+7x^2y^2-5xy^3+6y^4\\
3x^4-8x^3y\qquad\quad+7xy^3-12y^4\\
-6x^4\qquad+8x^2y^2-7xy^3\\
\qquad 14x^3y+2x^2y^2-xy^3-y^4\\
x^4+x^3y\qquad\quad+2xy^3+7y^4\\
\hline
2x^4+10x^3y+17x^2y^2-4xy^3
\end{array}$$

The student will observe from this example that in any column not only must the letters be the same, but also their *indices*.

Exercise II.

Add together

1. $3x+2y+7$
   $4x+12y+9$
   $7x-10y+4$
   $2x-8y-10$

2. $4a+2b+8$
   $2a-7b-13$
   $7a-15b-7$
   $3a+4b-11$

3. $2a^2+7a-9$
   $3a^2-4a-8$
   $7a^2+15a+10$
   $-4a^2+a+7$

4. $11x+10y+8z$
   $4x\qquad-5z$
   $3x+5y$
   $-x-y+z$
   $14y-7z$

5. $16x^3-2y^3+4z^3$
   $-x^3\qquad+5z^3$
   $-9y^3-7z^3$
   $-18x^3+4y^3$
   $-11x^3-y^3$

6. $4a^2\qquad-7$
   $2a^2+3a$
   $-6a+12$
   $7a^2-7a-7$
   $-a^2-a-1$

7. $15a+12b+6c-\ 4d-\ e$
$12a+\ b+7c+\ 3d-2e$
$-4a-\ 6b-8c+12d+4e$
$8a-\ 7b+4c-\ 9d-\ e$
$7a-\ 6b-9c-\ d+6e$

8. $4x^2+2y^2+\ 3xy+2x+8y$
$-x^2+3y^2+\ 2xy-4x+9y$
$5x^2-7y^2-10xy+3x-6y$
$10x^2-4y^2\qquad +4x-\ y$
$7y^2+\ 2xy\qquad +7y$

9. $16x^3\qquad -2x-12$
$-11x^3+7x^2\qquad +11$
$9x^3-\ x^2-\ x+\ 1$
$8x^3+7x+\ 5$
$-x^3\qquad -9x$

10. $11x^5-9x^4\qquad +\ 1$
$2x^5-3x^4-2x$
$x^4-\ x+12$
$x^5\qquad +4x-\ 3$
$8x^5+7x^4$

11. $3x^3+2x^2-5x+7$, $2x^3-12x^2+4x-10$,
$7x^3-14x^2+7x+9$, and x^3-x^2+2x-1.

12. $9x^4+2x^3+x^2+12x+1$, $x^4+9x^3+7x^2+x-9$,
$8x^4-7x^3-x^2-15x+10$,
and $2x^4+12x^3+2x^2+7x+6$.

13. $4a^2+12b^2+2c^2-3bc+4ca-2ab$,
$13a^2+b^2+12c^2+2bc-3ca+4ab$,
$a^2+9b^2-11c^2-2bc+12ca-2ab$,
and $2a^2-11b^2-c^2-bc-ca+2ab$.

14. $3a+2b+5c-6d$, $a+6b+10c-d$,
$-5a+b-4c-4d$, and $-5a+b-4c-4d$.

15. $-6a^2+12b^2+c^2+2bc-7ca+2ab$,
$a^2+5c^2-3bc+8ab$, and $4b^2-2c^2-3ca-10ab$.

16. $6x^2+7y^2+2z^2-5yz+3xz$, $2x^2-3z^2+4yz-2xz$, $7x^2+4y^2-3yz+4xz$, and $-2x^2+13yz$.

17. $7x^2-3z^2+2yz+12xz-4xy$, $9x^2+2y^2+4xz+2xy$, x^2-y^2-3xz, and $-4x^2+12y^2-13xz+12xy$.

18. $4a^2-2c^2-3b-14c$, $-2a^2+3b^2-4c^2+7b+11c$, $a^2-2b^2+11c^2+8a-9b$, and $3a^2-9a+4b+2c$.

19. $-15a^3-a^2b-2b^3$, $-4a^3-11a^2b-7ab^2+9b^3$, $6a^3-4ab^2+9b^3$, $-2a^3+6a^2b$, and $12a^2b-b^3$.

20. $4a^2+12bc$, $6b^2-7c^2+9bc+2ca$, $3a^2+2b^2+bc-3ca-7ab$, and $-8b^2-c^2-10bc$.

21. $3x^3-10y^3+5z^3-7yz$, $-x^3+4y^3-10z^3+3xy$, $z^3+11yz+8xz-2xy$, $4z^3-4yz+xz$, and $-2x^3+6y^2-9xz-xy$.

22. $2a^3-7a^2b+4b^3$, $-a^3+6ab^2+2c^3$, $5a^2b-2ab^2+b^3-c^3$, and $-4ab^2+2b^2c-7c^3$.

23. $1+3x+2x^3-x^5$, $4-2x^2-8x^3+7x^4$, $12x-4x^3+12x^5$, and $5x^2+7x^3-11x^5$.

24. $7x^2y+8xy^3-6y^4$, $12x^2y^2-4xy^3+5y^4$, $-8x^4+2x^3y+2x^2y^3-2y^4$, $3x^4-4x^3y+3xy^3+3y^4$, and $x^4-2x^2y^2$.

25. $11x^4-2xy^3$, $3x^3y-2x^2y^2+7xy^3-8y^4$, $8x^4-7x^3y+y^4$, $-12x^4+4x^2y+5x^2y^3$, and $-3x^2y^3-5xy^3+7y^4$.

26. $4a^5+12a^3-a-10$, $6a^4-a^3+2a^2-7$, $9a^3-3a^5+4a$, $11a-2a^4-a^5+9$, and $4a^3-a^4-5$.

27. $11x^3+14x^2y-7xz^2+z^3$, $x^3-2x^2y+3xy^2-7y^3+3y^2z-2z^3$, $4x^3+2y^3-11z^3+4yz^2-3xyz$, and $3y^3+10z^3-7y^2z+4xyz+4xy^2-4yz^2$.

28. $a^3-2b^3-11a^2b-4ab^2-bc^2$, $4b^3-abc+6c^3+9a^2b$
$-ac^2+ab^2$, $c^3-2b^3-a^3+4bc^2+3ab^2$,
and $2a^2b-3bc^2-7c^3+3abc$.

29. $a^3x+2a^2x^2-7x^4$, $5a^4-3a^3x^2+4ax^3-x^4$,
$5a^3x-2a^4+3x^4-2x^3-3$, $4x^3-2x^4-9-a^4-6a^3x$,
and $x^2-2x^3+7x^4-a^2x^3-1$.

30. $x^2yz-x^3z-2xz^3-7z^4$, $3y^4-x^4-2x^2y^2-3x^3z-4xy^3$,
$-2x^4-y^4-10xy^2z+4x^2z^2+4xz^3$,
$3x^4+8xyz^2-4x^2z^2+3y^4+7z^4-x^2yz$, $4x^3z+2y^4$
$+2x^2y^2+4xy^3+10xy^2z$, and $-7y^4-2xz^3-8xyz^2$.

CHAPTER III.

SUBTRACTION.

THE rule for Subtraction is, to place the expressions one under the other in the same way as in Addition; then change the sign of every term in the lower line, and proceed as in Addition. When the student has become more familiar with the process, he will be able to perform the subtraction without actually changing the signs of the lower line *on the paper*, but until then it will be unsafe for him to dispense with that help.

Ex. 1. From $4a+6b-7c+11d+4$ take $2a+5b+3c-4d+9$.

$$\begin{array}{r} 4a+6b-\ 7c+11d+4 \\ -2a-5b-\ 3c+\ 4d-9 \\ \hline 2a+\ b-10c+15d-5 \end{array}$$ Ans.

Ex. 2. From $6x^4+5x^3+3x+10$ take $5x^3+6x^2-3x+10$.

$$\begin{array}{r} 6x^4+5x^3\qquad +3x+10 \\ -5x^3-6x^2+3x-10 \\ \hline 6x^4\qquad -6x^2+6x \end{array}$$ Ans.

Ex. 3. From $7a^5+11a^4b+3a^3b^3-4ab^4-b^5$ take $5a^3b^2-4a^5+2a^2b^3+11a^4b+b^5$.

$$\begin{array}{r} 7a^5+11a^4b \qquad +3a^3b^3-4ab^4-b^5 \\ 4a^5-11a^4b-5a^3b^2-2a^2b^3 \qquad -b^5 \\ \hline 11a^5 \qquad -5a^3b^2+a^2b^3-4ab^4-2b^5 \end{array}$$ Ans.

Exercise III.

1. From $15a+13b+17c+12$
   take $4a+11b+16c+9$.
2. From $17a+6b+5c+10$
   take $3a+4b+4c+7$.
3. From $14a+12b+11c+15d$
   take $7a+11b+10c+d$.
4. From $5x^2+4xy+12y^2+9$
   take $4x^2+2xy+y^2+4$.
5. From $20x^3+17x^2y+15xy^2+2y^3$
   take $19x^3+15x^2y+9xy^2+2y^3$.
6. From $12a+15b-11c+5$
   take $9a-6b+10c-6$.
7. From $26a^2+16b^2+6c^2-12d^2$
   take $14a^2-16b^2+14d^2$.
8. From $11a^2-12b^2-19d^2$
   take $5a^2-3b^2-12c^2-21d^2$.
9. From $4a^2+b^2-16c^2$
   take $5a^2-11b^2-21c^2-21$.
10. From $a^3-12ab^2+15b^3$
    take $11a^3-4a^2b-12ab^2+16b^3$.
11. From $12a+15b+16c-8$ take $9a+4c+12$.
12. From $13x^2+12xy+11y^2$ take $11x^2-12xy+9y^2$.

13. From $4x^3-3x^2y+4xy^2+6y^3$ take $3x^3+2x^2y+4y^3$.

14. From $14a+3b+12c$ take $12a-3b+16c+2$.

15. From $8a^2+9b^2+7c^2+6bc$
take $4a^2-6b^2+8bc-6ab$.

16. From $13a^2-12b^2+6ca-2ab$
take $12a^2-2b^2+3c^2-2bc$.

17. From $14b^2+12c^2-7ca+9ab$
take $7a^2-3b^2-4c^2-3ca-2ab$.

18. From $2a^2+3b^2-2c^2+4bc-6ca$
take $15a^2+c^2-12bc-6ca+4ab$.

19. From $4x^3+z^3+2x^2y+3z^2x+12xyz$
take $3x^3-11y^3+3z^3-2x^2y-4z^2x+16xyz$.

20. From $15y^3+z^3-2xy^2+4yz^2-5z^2x$
take $6x^3-12y^3+4z^3-2xy^2+6z^2x$.

21. From $8a^3+3a^2-2a+7$ take $3a^3+7a-4$.

22. From $11x^4-2x^3+3x^2-8x$ take $3x^4-x^2+7x-14$.

23. From $4x^3-2xy^2$ take $2x^3-2x^2y+xy^2+y^3$.

24. From $12a^5-7a^4+3a^3-7$ take $2a^4-3a^3+2a-5$.

25. From $12x^4y+3x^2y^3+2xy^4-y^5$ take $2x^5-x^4y+7x^3y^2+2xy^4+y^5$.

26. From $4a^3b+7a^2b^2-2b^4$ take $2a^4-7a^3b^2-2ab^3+3b^4$.

27. From x^3+64 take x^3-4x^2+16x.

28. From x^5+3xy^4 take $x^5+2x^4y+3x^3y^2-2xy^4+y^5$.

29. From $16x^4+y^4$ take $8x^3y+4x^2y^2+2xy^3+y^4$.

30. From $8x^3y+4x^2y^2+2xy^3+y^4$ take $16x^4+y^4$.

31. From $4a^3$ take $3a^3+2a^2b+6ab^2-2b^3$.

32. From $7xy^3-2x^2y^2$ take $-4x^4+3x^2y^2-2xy^3+y^4$.

33. From $7ab^4 - 2b^5$ take $13a^5 + 6a^4b - 2a^3b^2$.

34. From $4x^4 - 3x^3y + xy^3$ take $4x^4 + 2x^2y^2 - xy^3 - y^4$.

35. From $6x^3 + 4x + 7$ take the sum of $2x^3 + 4x^2 + 9$ and $4x^3 - x^2 + 4x - 2$.

36. From $12x^5 + 5x$ take the sum of $4x^4 - 2x^3 + 3x^2 + 9$ and $6x^5 + 2x^4 + 2x^3 + 5x + 3$.

37. Take the sum of $6a^3 + 4a^2x + 3ax^2$, $2a^2x - ax^2 + x^3$ and $a^3x - 2ax^2 + 3x^3$, from $6a^3 + 2a^2x - 4x^3$.

38. Take the sum of

$a^4 - 2a^3x + x^4$, $2a^4 + 5a^3x + 3a^2x^2 - 5x^4$, and $6a^4 + a^3x + a^2x^2 - 4ax^3$ from $10a^4 - 4a^3x - 2a^2x^2 - ax^3$.

39. Take the sum of $x^3 + 3x - 2$, $2x^3 + x^2 - x + 5$,
and $4x^3 + 2x^2 - 7x + 4$, from the sum of $2x^3 + 9x$
and $5x^3 + 3x^2$.

40. From the sum of

$12x^5 + 4xy^4 + y^5$, $2x^5 - 4x^4y - xy^4 + 3y^5$,
and $6x^4y + 2x^2y^3 - 3xy^4$,
take the sum of $6x^5 + 2x^2y^3 - y^5$, $x^5 - 2x^4y + x^3y^2 + 2y^5$,
and $6x^5 + 4x^4y - 2x^3y^2 + 3y^5$.

CHAPTER IV.

MULTIPLICATION.

IN multiplying one quantity by another, the coefficients are multiplied, but the indices are added: thus

$$7x^3 \times 4x^5 = 28x^8,$$

where 7 and 4 are multiplied, but 3 and 5 are added.

Whenever a letter is written without an index, the index 1 is understood.

A simple example will indicate to the student the reason for *adding* the indices:—suppose we have to multiply x^4 by x^3; we know that

$$x^4 = x \times x \times x \times x,$$

and $$x^3 = x \times x \times x,$$

hence $$x^4 \times x^3 = x \times x \times x \times x \times x \times x \times x = x^7.$$

The rule with regard to signs is as follows:

$$+x \times +x = +x^2; \quad +x \times -x = -x^2,$$

$$-x \times -x = +x^2; \quad -x \times +x = -x^2,$$

which is briefly expressed by saying that like signs have + as their product, and unlike have −.

Exs. $4x^3 \times 3x^5 = 12x^7$; $12x^{12} \times 6x^6 = 72x^{18}$;

$5x^9 \times 3x = 15x^{10}$; $11x^2y^3 \times 4x^4y^7 = 44x^6y^{10}$;

$10x^2y^2z \times -x^2y = -10x^5y^4z$; $-7abc \times 5a^2b^3x = -35a^3b^4cx$;

$-12a^5c^2x \times -7b^3cxy = 84a^3b^3c^3x^2y$; $-6ab^2x \times -axz^2 = 6a^2b^2x^2z^2$.

When the multiplicand consists of several terms, each term in it must be multiplied separately.

Exs. Multiply $2x^3 - 3x^2y + 4xy^2$ by $4axy^2$.

$$\begin{array}{l} 2x^3 - 3x^2y + 4xy^2 \\ 4axy^2 \\ \hline 8ax^4y^2 - 12ax^3y^3 + 16ax^2y^4 \end{array}$$ Ans.

Multiply $4a^4x - 5a^3x^2 - ax^4 + 2x^5$ by $-11ax^2z^2$.

$$\begin{array}{l} 4a^4x - 5a^3x^2 - ax^4 + 2x^5 \\ -11ax^2z^2 \\ \hline -44a^5x^2z^2 + 55a^4x^3z^2 + 11a^2x^6z^2 - 22ax^7z^2 \end{array}$$ Ans.

When the multiplier as well as the multiplicand consists of more terms than one, the whole of the multiplicand must be multiplied by each term of the multiplier, and the results arranged under each other and added.

Exs. Multiply $2x^2+3xy-y^2$ by $3x-2y$.

$$\begin{array}{r} 2x^2+3xy-y^2 \\ 3x\ -2y \\ \hline 6x^3+9x^2y-3xy^2 \\ \quad -4x^2y-6xy^2+2y^3 \\ \hline 6x^3+5x^2y-9xy^2+2y^3 \end{array}$$ Ans.

Multiply $3x^4-x^3-1$ by $2x^4-3x^2+7$.

$$\begin{array}{r} 3x^4-\ x^3-\ 1 \\ 2x^4-\ 3x^3+\ 7 \\ \hline 6x^8-\ 2x^6-\ 2x^4 \\ \quad -\ 9x^6+\ 3x^4+3x^3 \\ \qquad\qquad 21x^4-7x^3-7 \\ \hline 6x^8-11x^6+22x^4-4x^3-7 \end{array}$$ Ans.

To find the square of an expression, multiply it by itself: to find its cube, multiply it into itself 3 times: to find its fourth power, 4 times; and so on.

Exercise IV.

Multiply

1. $2x^2$ by $3x^3$.
2. $-4x^3$ by $2x^4$.
3. $5a^6$ by $-3a^3$.
4. $-6a^2$ by $-7a^5$.
5. $4a^3b^2$ by $7a^4b^3$.
6. $-12x^2y^3$ by $4x^3y^4$.
7. $6x^3z^2$ by $-4x^3z^7$.
8. $-8a^5b^3c^3$ by $-3a^2b^5c^3$.
9. $5ab^2$ by $3a^2b$.
10. $7xy^3z^2$ by $4x^2yz^2$.

11. $5ab^2$ by $3a^2$.
12. $-4a^2x^3$ by $5x^5$.
13. $6x^2y^3$ by $-3x^4z^3$.
14. $5a^4b$ by $2b^3c^3d$.
15. $-7a^2bx^3$ by $4b^2xz$.
16. $3ax^3z$ by $5b^2cz$.
17. $2a^2b^2$ by a^3b^4.
18. x^2y^3 by $-2x^4y^5$.
19. x^2y^4 by $-x^3y^4$.
20. $3axy^2$ by xy^3z.
21. $3x^3-5x^2+3x$ by $5x^2$.
22. $7x^4-6x^3-9x$ by $-8x^3$.
23. $5a^5-7a^3-8a+5$ by $6a^3$.
24. $b^4+8b^3-7b^2+11b-5$ by $-7b$.
25. $3a^2-5a-2$ by $6b$.
26. $8a^4-7a^3x+6a^2x^2-5ax^3+7x^4$ by $3a^2x^2$.
27. $8x^5y^{10}-7x^3y^6+4x^2y^4-3xy^2$ by $-2xy^2$.
28. $8a^5-7a^4b+6a^3b^2-7a^2bc+9a^2b^2c+c^5$ by $3a^2b^3$.
29. $9x^2y^2z^3-7xy^3z^2-4xz^5$ by $-5xy^2z^3$.
30. $7a^5b^2-9a^5c^2+12a^3b^2c^2-7b^4c^3$ by $4c^6$.
31. $2a+b$ by $2a+3b$.
32. $3x+2y$ by $4x+3y$.
33. $5y-7z$ by $4y-2z$.
34. $a+b$ by $a-b$.
35. x^2+y^2 by x^2-y^2.
36. $3a^2-4a+5$ by $2a+5$.
37. $5x^2+4x-3$ by $3x+4$.
38. $2x^2-3xy+y^2$ by $2x+3y$.
39. $4x^2-2ax+a^2$ by $2x+a$.
40. $7a^2-5a+3$ by $3a-2$.
41. $3x^2-7x+1$ by $2x-5$.

42. x^2+ax+a^2 by $x-a$.

43. $2x^2-4bx+3b^2$ by $2x-3b$.

44. $4x^3-3x^2-2x+5$ by $3x-5$.

45. $3a^3+5a^2-7a+3$ by $2a-3$.

46. $5a^3-3a^2b-5ab^2+4b^3$ by $3a+5b$.

47. $2y^3+3by^2-2b^2y-3b^3$ by $4y-3b$.

48. $x^4-ax^3+a^2x^2-a^3x+a^4$ by $x+a$.

49. $3x^4-2x^3y+4x^2y^2-7xy^3+16y^4$ by $2x-3y$.

50. a^2b^2+ab+1 by $ab-1$.

51. $2x^2-3x+4$ by $3x^2-2x-5$.

52. $4x^2+2x+3$ by x^2-3x-4.

53. $3a^2-3a-1$ by $2a^2+4a+3$.

54. $7y^2-4y+3$ by $2y^2-4y-3$.

55. $y^2+4y+16$ by $y^2-4y+16$.

56. b^4-2b^2+1 by b^4+2b^2+1.

57. a^4+3a^2-2 by a^4-4a^2+1.

58. x^6-4x^3+4 by x^6+4x^3+4.

59. $2y^6+3y^3+5$ by $3y^6-2y^3-3$.

60. $2a^2-3ab+3b^2$ by $3a^2-4ab-2b^2$.

61. $4x^2-3ax+a^2$ by $2x^2-4ax-5a^2$.

62. $5x^2+4xy-2y^2$ by $2x^2-3xy-4y^2$.

63. $2y^4-3b^2y^2+b^4$ by $3y^4-2b^2y^2-2b^4$.

64. $x^3-x^2y+xy^2-y^3$ by $x^2+2xy+y^2$.

65. $a^3+2a^2b+4ab^2+8b^3$ by $a^2-4ab+4b^2$.

66. $2x^4-3x+4$ by $2x-3$.

67. $3a^5-2a^3+3a-1$ by $3a+2$.

68. $2x^4-3xy^3+y^4$ by $3x-y$.

69. $x^5-3ax^4+2a^5$ by $2x+3a$.

70. x^5-2x^3+3 by $2x-5$.

71. $x^6-a^2x^4+a^4x^2-2a^6$ by $2x-3a$.

72. x^3-2x+1 by x^2-4.

73. x^5-2x^2+3 by $2x^3-3x-1$.

74. $a^5-2a^3b^3-b^5$ by $a^3-ab^2+b^3$.

75. $a^6-a^2bc+b^2c^2$ by a^4-bc.

76. $a^2+b^2+c^2-ab-ac-bc$ by $a+b+c$.

77. $a^2+b^2+c^2+ab+ac-bc$ by $a-b-c$.

78. $x^3+4y^2+3z^2$ by $x^2-2y^2-3z^2$.

79. $a^2+b^2+9-3a+3b+ab$ by $a-b+3$.

80. $4x^2+9y^2+z^2-6xy-2xz-3yz$ by $2x+3y+z$.

Find the square of

81. $a+b$.

82. $2x+3y$.

83. $3x-4a$.

84. $4y-5b$.

85. $a+b+c$.

86. x^2-3x+4.

Find the cube of

87. $a+b$.

88. $2x-3y$.

Find the fourth power of

89. $x-y$.

90. $2a+5b$.

CHAPTER V.

Division.

To divide one simple expression by another, divide one coefficient by the other, and take the difference between the indices of the same letters: thus, $20x^5 \div 2x^2 = 10x^3$; $75x^4y^2z^3 \div 15xz^3 = 5x^3y^2z$.

The rule of signs is the same as in Multiplication.

Exs. $100a^5b \div 20a^3b = 5a^2$; $45x^2y^3z^4 \div -9yz^3 = -5x^2y^2z$;
$-28a^4x^3y \div 7a^3y = -4ax^3$; $-20a^4x^2y^3z \div -4a^3y^3 = 5ax^2z$.

When there are more terms than one in the dividend, and only one in the divisor, each term is divided separately

Exs. Divide $12x^4y^3 - 20x^3y^3 + 24x^2y^4 - 4xy^5$ by $4xy^2$.

$4xy^2 \mid 12x^4y^3 - 20x^3y^3 + 24x^2y^4 - 4xy^5$

$3x^3 - 5x^2y + 6xy^2 - y^3$. Ans.

Divide $56ab^3c^3 + 88a^2b^3c^3 - 8a^3b^4c^4 - 24a^4b^4c^3$ by $-8ab^3c^3$.

$-8ab^3c^3 \mid 56ab^3c^3 + 88a^2b^3c^3 - 8a^3b^4c^4 - 24a^4b^4c^3$

$-7c^5 - 11abc^3 + a^2b^2c + 3a^3b^3$. Ans.

When the divisor consists of more terms than one, the division is performed as follows:—divide the first term of the dividend by the first term of the divisor, placing the result as the first term of the quotient or answer: multiply the whole of the divisor by this quotient, and place what you obtain under the dividend: subtract, and divide the first term of the remainder by the first term of the divisor, placing the result in the quotient as before; repeat this process until there is no remainder or until the divisor will no longer divide the remainder.

Ex. 1. Divide $3x^2+4xy+y^2$ by $x+y$.

$$\begin{array}{r|l} x+y)\ 3x^2+4xy+y^2 & (3x+y. \quad \text{Ans.} \\ 3x^2+3xy & \\ \hline xy+y^2 & \\ xy+y^2 & \\ \hline \end{array}$$

In this example, we first divide $3x^2$ by x, and place the result $3x$ in the quotient: we then multiply the divisor $x+y$ by $3x$ and place the product $3x^2+3xy$ below the dividend; by subtraction we obtain the remainder $xy+y^2$, and divide the first term of this (xy) by the first term (x) of the divisor: the result y we add to the quotient and multiplying the whole divisor by it, place the product under the former remainder and subtract: there will then be no remainder.

Ex. 2. Divide $12x^5+5x^4y-12x^3y^2+x^2y^3+xy^4-2y^5$ by $3x^3+2x^2y-xy^2+y^3$.

$$\begin{array}{r|l} 3x^3+2x^2y-xy^2+y^3) & \\ 12x^5+5x^4y-12x^3y^2+x^2y^3+xy^4-2y^5 & (4x^2-xy-2y^2. \quad \text{Ans.} \\ 12x^5+8x^4y-\ 4x^3y^2+4x^2y^3 & \\ \hline -3x^4y-\ 8x^3y^2-3x^2y^3+xy^4-2y^5 & \\ -3x^4y-\ 2x^3y^2+\ x^2y^3-xy^4 & \\ \hline -\ 6x^3y^2-4x^2y^3+2xy^4-2y^5 & \\ -\ 6x^3y^2-4x^2y^3+2xy^4-2y^5 & \\ \hline \end{array}$$

Ex. 3. Divide $9a^2-4b^2$ by $3a+2b$.

$$\begin{array}{r|l} 3a+2b)\ 9a^2-4b^2 & (3a-2b. \quad \text{Ans.} \\ 9a^2+6ab & \\ \hline -6ab-4b^2 & \\ -6ab-4b^2 & \\ \hline \end{array}$$

Ex. 4. Divide $10a^3+8a+12a^4-4-26a^2$ by $1+2a^2-3a$.

In this question, the expressions must be rearranged according to descending powers of a.

$2a^2-3a+1)\,12a^4-26a^3+10a^2+8a-4\,(6a^2-4a-4.$ Ans.

$12a^4-18a^3+6a^2$

$-8a^3+4a^2+8a$

$-8a^3+12a^2-4a$

$-8a^2+12a-4$

$-8a^2+12a-4$

Ex. 5. Divide $27a^3-b^3+8c^3+18abc$ by $3a-b+2c$.

$3a-b+2c)\,27a^3+18abc-b^3+8c^3\,(9a^2+3ab-6ac+b^2+2bc+4c^2.$ Ans.

$27a^3-9a^2b+18a^2c$

$9a^2b-18a^2c+18abc-b^3+8c^3$

$9a^2b-3ab^2+6abc$

$-18a^2c+3ab^2+12abc-b^3+8c^3$

$-18a^2c+6abc-12ac^2$

$3ab^2+6abc+12ac^2-b^3+8c^3$

$3ab^2-b^3+2b^2c$

$6abc+12ac^2-2b^2c+8c^3$

$6abc-2b^2c+4bc^2$

$12ac^2-4bc^2+8c^3$

$12ac^2-4bc^2+8c^3$

Exercise V.

Divide

1. $8x^5$ by $2x^3$.
2. $15a^7$ by $5a^3$.
3. $-14y^5$ by $7y^3$.
4. $21y^9$ by $-7y^2$.
5. $-12x^{10}$ by $-3x^3$.
6. $-8x^7$ by $-4x$.
7. $12a^9b^3$ by $4a^5b$.
8. $13a^5b^2c$ by $-a^2b$.
9. $-a^3b^4$ by $-ab$.
10. $3x^6yz^3$ by $-x^2z^2$.

11. $2x^4-3x^3+4x$ by x.
12. $3x^6-12x^3+15x^2$ by $3x^2$.
13. $10y^{13}-15y^8+20y^6$ by $-5y^4$.
14. $-18a^4+27a^3-9a^2$ by $-9a^2$.
15. $12b^6-10b^3+8b^2$ by $2b^2$.
16. $21a^6b^3-14a^4b^2-7a^2b$ by $7a^2b$.
17. $-x^6-13x^5y+12x^4y^2-10x^3y^3$ by $-x^3$.
18. $-4x^8-12x^6y^2-20x^4y^4$ by $2x^3$.
19. $18a^6b^3-12a^4b^4-24a^2b^6$ by $6a^2b^2$.
20. $21a^3b^2c-14a^2b^5c^2+7a^4b^6$ by $7a^2b^2$.
21. $x^2+12x+20$ by $x+2$.
22. $y^2+14y+40$ by $y+4$.
23. $x^2-x-132$ by $x-12$.
24. $x^2-3x-70$ by $x+7$.
25. $2a^2-14a^2+24$ by $2a-6$.
26. $3a^2-26a+35$ by $3a-5$.
27. $2x^2+15x-50$ by $2x-5$.
28. $3x^2-5x-28$ by $3x+7$.
29. $3x^2-11xy+6y^2$ by $3x-2y$.
30. $12a^2-ab-b^2$ by $4a+b$.
31. $x^3+3x^2-23x^2+35$ by $x+7$.
32. $x^3-12x^2+27x+40$ by $x-5$.
33. $2a^3-7a^2-3a+18$ by $2a+3$.
34. $3a^3+17a^2-43a+20$ by $3a-4$.
35. $4y^3-10y^2-62y-30$ by $2y+5$.
36. $3x^3-2x^2y-5xy^2+4y^3$ by $3x+4y$.

37. $3x^3-16ax^2+28a^2x-16a^3$ by $3x-4a$.

38. $6x^3-x^2y-23xy^2-12y^3$ by $2x+3y$.

39. $2x^4-17x^3+41x^2-29x+28$ by $2x-7$.

40. $8x^4+2x^3y-31x^2y^2+40xy^3-25y^4$ by $4x-5y$.

41. $6x^4+23x^3+42x^2+41x+20$ by $3x^2+4x+5$.

42. $6x^4-23x^3+32x^2-9x-18$ by $2x^2-5x+6$.

43. $20x^4-33x^3+72x^2-35x+30$ by $4x^2-5x+10$.

44. $2y^4-16y^3+2y^2+92y+48$ by $y^2-5y-12$.

45. $3a^4-23a^3-5a^2+80a+50$ by $a^2-6a-10$.

46. $x^4-8x^3y+21x^2y^2-16xy^3-7y^4$ by $x^2-5xy+7y^2$.

47. $x^4-9ax^3+12a^2x^2+35ax^3+15a^4$ by $x^2-4ax-3a^2$.

48. $4a^4-16a^3b-4a^2b^2+40ab^3+25b^4$ by $2a^2-4ab-5b^2$.

49. $4y^4-15by^3+26b^2y^2-23b^3y+8b^4$ by $4y^2-7by+8b^2$.

50. $5x^4-14x^3y+31x^2y^2-22xy^3+12y^4$ by $5x^2-4xy+3y^2$.

51. $6a^4+21a^3b+31a^2b^2+27ab^3-5b^4$ by $3a^2+6ab-b^2$.

52. $2a^4-2a^3b-3a^2b^2+4ab^3+5b^4$ by $2a^2-6ab+5b^2$.

53. $6x^4-7ax^3-36a^2x^2+17a^3x+10a^4$ by $3x^2+4ax-5a^2$.

54. $27x^4-3x^3y-25x^2y^2+25xy^3-6y^4$ by $9x^2-7xy+2y^2$.

55. $8x^4-2x^3y-21x^2y^2-2xy^3+y^4$ by $4x^2+5xy-y^2$.

56. $16x^4-46x^3y-21x^2y^2+5xy^3-10y^4$
by $8x^2-3xy+2y^2$.

57. $14a^4-45a^3b+16a^2b^2-15ab^3-18b^4$
by $7a^2-5ab+6b^2$.

58. $9x^4-9ax^3-13a^2x^2-16a^3x-6a^4$ by $3x^2-5ax-3a^2$.

59. $x^5-10x^4+17x^3+41x^2+7x-6$ by x^2-5x-6.

60. $x^5-5x^4y+11x^3y^3-14x^2y^3+9xy^4-2y^5$ by $x^2-3xy+2y^2$.

61. a^2-b^2 by $a+b$.

62. $4x^2-9y^2$ by $2x-3y$.

63. a^3+b^3 by $a+b$.

64. a^3-b^3 by $a-b$.

65. $27x^3-8a^3$ by $3x-2a$.

66. $16a^4-81b^4$ by $2a-3b$.

67. $16a^4-81b^4$ by $2a+3b$.

68. x^6-8y^6 by x^2-2y^2.

69. x^3+64 by $x^2-4x+16$.

70. y^3-27 by y^2+3y+9.

71. $x^4+x^2y^2+y^4$ by x^2-xy+y^2.

72. a^4+16a^2+256 by $a^2+4a+16$.

73. $a^8+a^4b^4+b^8$ by $a^4-a^2b^2+b^4$.

74. $x^5-x^3y^2-x^2y^3+y^5$ by x^2+xy+y^2.

75. $a^5-4a^3b^2-8a^2b^3+32b^5$ by a^2-4b^2.

76. x^6-y^6 by x^2-xy+y^2.

77. a^6-64 by a^2+2a+4.

78. $x^4+21x^2-7-8x^3-16x$ by x^2-5x+7.

79. $5x^4+31x^2-22x+12-14x^3$ by $5x^2+3-4x$.

80. $2x^4+4x+5-2x^3-5x^2$ by x^2+2x+1.

81. $9x^4-9x^3-6-16x-13x^2$ by $3x^2+2+2x$.

82. $x^8+x^4y^4+y^8$ by x^2+xy+y^2.

83. x^6-y^6 by $x^3+2x^2y+2xy^2+y^3$.

84. $x^6-x^4y^2+3x^3y^3-x^2y^4+y^6$ by x^2-xy+y^2.

85. $a^5-a^3y^2-a^2y^3+y^5$ by $a^3+2a^2y+2ay^2+y^3$.

86. $a^2+2ab+2ac+b^2+2bc+c^2$ by $a+b+c$.

87. $a^3-3abc+b^3+c^3$ by $a+b+c$.

88. $a^3-8b^3+c^3+6abc$ by $a-2b+c$.

89. $a^2-b^2+2bc-c^2$ by $a+b-c$.

90. $a^4+b^4+c^4-2a^2b^2-2a^2c^2-2b^2c^2$ by $a^2+2ab+b^2-c^2$.

CHAPTER VI.

Square Root.

The Square Root of any quantity is that quantity which when multiplied by itself will produce the given quantity.

Thus $5x^3$ is the square root of $25x^6$
$8x^2y^4z$ ..... $64x^4y^8z^2$
$a+b$ ..... $a^2+2ab+b^2$
$3x-2y^2$ ..... $9x^2-12xy^2+4y^4$.

The Square Root of an expression of only one term is found by inspection.

To find the square root of an expression containing three terms, first place in the quotient the square root of the first term: then multiply this root by itself and place the result under the given expression, and subtract: a divisor is now found by doubling the term in the quotient: divide this into the first term of the dividend, and place the result *both in the quotient and divisor:* then multiply

both terms of the divisor by the last term of the quotient, and place the result under the dividend.

Ex. 1. Find the square root of $25a^2 + 20ab + 4b^2$.

We first find $5a$, the square root of $25a^2$, and place it in the quotient. Then we double $5a$, and make $10a$ the first term of the divisor. This, divided into $20ab$, gives $2b$, which is placed both in the quotient and divisor: lastly we multiply $10a + 2b$ by $2b$, and write the product under the dividend. The quotient $5a + 2b$ is the square root.

$$\begin{array}{r|l} 25a^2 + 20ab + 4b^2 & (5a + 2b \\ \hline 25a^2 & \\ \hline 10a + 2b & 20ab + 4b^2 \\ & 20ab + 4b^2 \end{array}$$

If the given expression consist of more than 3 terms, there will be a remainder after the second term of the answer has been obtained. In this case a new divisor must then be formed by doubling both terms of the quotient, and, as before, the new term of the root must be added both to the quotient and the divisor: if there still be a remainder, the same process is again repeated, care being taken always to form the new divisor by doubling the terms already in the quotient, and to add to the quotient and divisor the new term of the root.

Ex. 2. Find the square root of $x^4 - 4x^3 + 12x^2 - 16x + 16$.

$$\begin{array}{r|l} x^4 - 4x^3 + 12x^2 - 16x + 16 & (x^2 - 2x + 4. \quad \text{Ans.} \\ \hline x^4 & \\ \hline 2x^2 - 2x & -4x^3 + 12x^2 - 16x + 16 \\ & -4x^3 + 4x^2 \\ \hline 2x^2 - 4x + 4 & 8x^2 - 16x + 16 \\ & 8x^2 - 16x + 16 \end{array}$$

Ex. 3. Find the square root of x^4-4x^3+8x+4.

$$x^4-4x^3+8x+4 \;(x^2-2x-2. \quad \text{Ans.}$$

$$\begin{array}{r|l} x^4 & \\ \hline 2x^2-2x & -4x^3+8x+4 \\ & -4x^3+4x^2 \\ \hline 2x^2-4x-2 & -4x^2+8x+4 \\ & -4x^2+8x+4 \\ \hline \end{array}$$

The Fourth Root of any expression is the square root of its square root; the Eighth Root is the square root of its fourth root.

Exercise VI.

Find the square root of

1. $9a^2$. 2. $16a^4b^2$. 3. $36a^6b^4c^2$.
4. $81a^8b^6c^4d^2$. 5. $144x^2y^6z^4$. 6. $100x^{12}y^8z^{16}$.
7. $64a^2x^6y^{10}$. 8. $a^6b^8x^2y^{12}$. 9. $25a^4b^6x^{10}y^8z^{14}$.
10. $121a^2b^4c^6x^8y^{10}z^{20}$. 11. $x^2+8x+16$.
12. x^4+20x^2+100. 13. a^6+12a^3+36.
14. $9a^2+12a+4$. 15. $16a^6x^3+40a^4x+25a^2$.
16. $121x^8y^4+66x^6y^2+9x^4$. 17. $25x^4-70x^2+49$.
18. $36a^6-60a^3+25$. 19. $100a^8-60a^4+9$.
20. $144a^2b^4-216ab^2+81$. 21. $49x^4z^2-42a^2x^2z+9a^4$.
22. $25a^2b^4-20ab^2c^3+4c^6$.
23. $49x^4-154x^2yz^2+121y^2z^4$.
24. $4-28x+49x^2$. 25. $1-6ax^2+9a^2x^4$.

26. $\dfrac{16-24x+9x^2}{9-30x^2+25x^4}$. 27. $\dfrac{1-18x^2+81x^4}{9-30xy^2+25x^2y^4}$.

28. $\dfrac{25a^2-30axyz+9x^2y^2z^2}{49c^2-28cz^2+4z^4}$.

29. $\dfrac{x^2-10x^6y^3+25x^{10}y^4}{16x^2-56x^4y^3+49x^6y^6}$.

30. $\dfrac{9a^6x^{10}+42a^3x^6+49x^2}{25a^2x^4-80ax^4+64x^6}$.

31. $1+4x+8x^2+8x^3+4x^4$.

32. $x^4-4x^3+2x^2+4x+1$.

33. $x^4+2x^3+3x^2+2x+1$.

34. $16x^4+16x^3+24x^2+4x+1$.

35. $x^4-6x^3+27x^2-54x+81$.

36. $25a^4+30a^3-a^2-6a+1$.

37. $9a^8-42a^6+61a^4-28a^2+4$.

38. $121+66xy-101x^2y^2-30x^3y^3+25x^4y^4$.

39. $16-24ab^2+25a^2b^4-12a^3b^6+4a^4b^8$.

40. $81a^8+18a^6b-71a^4b^2-8a^2b^3+16b^4$.

41. $4x^4+24x^3-108x+81$.

42. $64-192x^2+216x^6+81x^8$.

43. $x^4-14x^3+31x^2+126x+81$.

44. $x^4-6x^3+x^2+24x+16$.

45. $x^8-4x^5-14x^3+36x+81$.

46. $x^{12}-18x^9+20x^6-18x^3+1$.

47. $16-24b+9b^2+8c-6bc+c^2$.

48. $64-64x+16x^2+64y^2-32xy^2+16y^4$.

49. $1+2x+3x^2+4x^3+3x^4+2x^5+x^6$.

50. $a^{12}+4a^{10}x+2a^8x^2-2a^6x^3+5a^4x^4-2a^2x^5+x^6$.

51. $x^6+4x^5y+2x^4y^2+9x^3y^4-4xy^5+4y^6$.

52. $64a^{12}-128a^{10}b+160a^8b^2-160a^6b^3+100a^4b^4-48a^2b^5+16b^6$.

53. $4-8xy^3+36x^2y^4-64x^3y^6+96x^4y^8-128x^5y^{10}+64x^6y^{12}$.

54. $x^{16}-2x^{14}y+3x^{12}y^2-4x^{10}y^3+5x^8y^4-4x^6y^5+3x^4y^6-2x^2y^7+y^8$.

55. $a^8-4a^7b+4a^6b^2+6a^5b^3-14a^4b^4+4a^3b^5+9a^2b^6-6ab^7+b^8$.

56. $x^{10}-2x^8y^3+5x^6y^4-6x^5y^5-4x^4y^6+6x^3y^7+4x^2y^8-12xy^9+9y^{10}$.

Find the fourth root of

57. $a^4+4a^3b+6a^2b^2+4ab^3+b^4$.

58. $16x^4+96x^3+216x^2+216x+81$.

59. $81x^8-108x^6y+54x^4y^2-12x^2y^3+y^4$.

60. $16a^4-32a^3b^2+24a^2b^4-8ab^6+b^8$.

61. $16-160a+600a^2-1000a^3+625a^4$.

62. $1+16x^2y+96x^4y^2+256x^6y^3+256x^8y^4$.

Find the eighth root of

63. $a^8-8a^7b+28a^6b^2-56a^5b^3+70a^4b^4-56a^3b^5+28a^2b^6-8ab^7+b^8$.

64. $x^8-16x^7+112x^6-448x^5+1120x^4-1792x^3+1792x^2-1024x+256$.

CHAPTER VII.

SQUARE ROOT OF NUMBERS.

THE Square Root of a number is found by the same method as that of an Algebraical expression. First mark off every two figures, beginning at the right hand, so as to leave either one or two figures at the beginning of the number. Take the square root of the figure or figures thus left, or if there be no exact square root, take the root of the square next below the number formed by them. Then place the square of this root under the number and subtract; bring down the next two figures, and treat the whole number thus obtained as a dividend; form a divisor for it by doubling the figure in the quotient, and divide this into the dividend excepting the last figure; the number of times it goes into it must be placed at the end both of *the quotient and the divisor:* again subtract, and bring down two more figures, and repeat this process until there is no remainder, taking care to form a new divisor each time by doubling the quotient already obtained.

EX. 1. Find the square root of 7918596.

7'91'85'96 (2814. Ans.

	4
48	391
	384
561	785
	561
5624	22496
	22496

In this example, after marking off every two figures, we

have left 7, the nearest square root to which is 2, and this we place in the quotient. We multiply 2 by itself, place 4 under 7, subtract, and then bring down 91. The divisor is formed by doubling the quotient 2. Then dividing 4 into 39 (*not* 391) we obtain 8, which we add both to the quotient and divisor; it will be observed that we cannot take 9 instead of 8, because by adding 9 to the divisor, we should get a product too large for the dividend. We now have a remainder 7, and we bring down 85; we double the quotient 28, and have 56 as the new divisor. Divide as before, omitting the last figure in the dividend in determining the quotient: we then obtain a remainder 224, and bringing down the figures 96, we find a quotient 4 and no remainder. Thus we have 2814 as the square root.

Ex. 2. Find the square root of 1863821584.

18'63'82'15'84 (43172. Ans.

16

83	263
	249
861	1482
	861
8627	62115
	60389
86342	172684
	172684

When part of the given number is a decimal fraction, the marking off must be begun, as before, at the end of the whole number, so that the figures after the point may always be brought down by themselves: the decimal point must be placed in the quotient when you come to it in the dividend.

If the whole of the number be a decimal fraction, place the point in the quotient at once, and always begin with two figures of the given number, adding 0 if the given number consist of only one significant figure. In some questions the square root does not terminate, but in these it is generally stated to how many figures the answer is to be taken. If the decimal begin with 0s, place one 0 in the quotient for every two 0s in the given number.

Ex. 3. Find the square root of 291·0436.

2′91·0436 (17·06 Ans.
1

27	191
	189
3406	20436
	20436

Ex. 4. Find the square root of ·9 to four places of decimals.

·90 (·9486 &c. Ans.
81

184	900
	736
1888	16400
	15104
18966	129600
	113796

Ex. 5. Find the square root of ·04.

·04 (·2 Ans.
·04

Ex. 6. Find the square root of ·0000075 to five places of decimals.

·0000075 (·00273 &c. Ans.

	4
47	350
	329
543	2100
	1629

If the given number involve a recurring decimal, it is generally best to reduce it to a vulgar fraction, making it an improper fraction if there be a whole number with the fraction: the resulting square root must be expressed as a decimal.

Ex. 7. Find the square root of $\cdot00\dot{4}$.

$\cdot00\dot{4}=\frac{4}{900}$; of which the square root is $\frac{2}{30}=\cdot0\dot{6}$. Ans.

Ex. 8. Find the square root of $53\cdot\dot{7}$.

$53\cdot\dot{7}=53\frac{7}{9}=\frac{484}{9}$; the square root of which is $\frac{22}{3}=7\cdot\dot{3}$. Ans.

Exercise VII.

*** No answer need be carried more than five places beyond the decimal point.

Find the square root of

1.	625.	2.	256.	3.	729.	4.	441.
5.	821.	6.	1225.	7.	2209.	8.	2601.
9.	5184.	10.	8649.	11.	3481.	12.	5329.
13.	3844.	14.	12769.	15.	14884.	16.	94249.

17. 29929. 18. 46656. 19. 263169.
20. 383161. 21. 495616. 22. 659344.
23. 942841. 24. 4376464. 25. 5322249.
26. 25623844. 27. 49042009. 28. 44352·36.
29. 6090·2416. 30. 493365·76.
31. 2250300·01. 32. 4·44155625.
33. 332·150625. 34. 18179990·44.
35. 56875730·56. 36. 49·871844.
37. 371246·49. 38. 635·191209.
39. 216·15174441. 40. 494210406001.
41. ·25. 42. ·0081. 43. ·008. 44. ·6.
45. ·000004. 46. ·00004. 47. ·08. 48. ·5.
49. ·1. 50. 1000. 51. 2·5. 52. 325·4.
53. 4·9. 54. 50·701. 55. 81·25. 56. ·017.
57. 7·1. 58. ·00004. 59. ·217. 60. ·00284.

CHAPTER VIII.

BRACKETS.

WHEN the sign + stands before a bracket, the bracket may be removed without making any change in the terms within it.

When the sign − stands before a bracket, the bracket may be removed, if the sign of every term within it be changed from + to −, or from − to +.

When one bracket is placed within another, the inner bracket must be removed first; similarly, if there are several brackets, one within another, the inmost must be removed first, then that next to it, and so on; the bracket which contains all the others being removed last of all.

Sometimes a line or bar, called a *vinculum*, is placed above some of the terms; it is used in the same manner as a bracket, and the same rules apply to its removal.

When the given expression has been entirely cleared of brackets, it must be further simplified by adding or subtracting all the terms of one kind.

Ex. 1. $a+2b+3c+(2a-4b+7c)-(a+2b-c)$

$=a+2b+3c+2a-4b+7c-a-2b+c$

$=2a-4b+11c.$ Ans.

Ex. 2. Simplify

$$2x^3+3x^2-(x^2+2x)-\{x^3-(3x^2+7x-4)+11x\}.$$

In this example, we must at first retain the bracket which contains another; hence we have

$$2x^3+3x^2-x^2-2x-\{x^3-3x^2-7x+4+11x\}$$

$$=2x^3+3x^2-x^2-2x-x^3+3x^2+7x-4-11x$$

$$=x^3+5x^2-6x-4.$$ Ans.

Ex. 3. $2x^3+\{4x^2-(7x^3-2x)\}$

$$-[8x^2-4x-\{5x^3+(6x^2-\overline{4x-7})-11x\}]$$

$$=2x^3+\{4x^2-7x^3+2x\}-[8x^2-4x-\{5x^3+(6x^2-4x+7)-11x\}]$$

$$=2x^3+4x^2-7x^3+2x-[8x^2-4x-\{5x^3+6x^2-4x+7-11x\}]$$

$$=2x^3+4x^2-7x^3+2x-[8x^2-4x-5x^3-6x^2+4x-7+11x]$$

$$=2x^3+4x^2-7x^3+2x-8x^2+4x+5x^3+6x^2-4x+7-11x$$

$$=2x^2-9x+7.$$ Ans.

If a figure or letter stand before a bracket, each term in the bracket must be multiplied by it when the bracket is removed. If there are more brackets than one in the question, care must be taken to multiply by the coefficient, only when its own bracket is being removed.

Ex. 4. $5(a^2-ax-2y^2)-a(a-x)+2y(2x+5y)-4x(x-a+y)$

$=5a^2-5ax-10y^2-a^2+ax+4xy+10y^2-4x^2+4ax-4xy$

$=4a^2-4x^2=4(a^2-x^2)$. Ans.

Since the result $4a^2-4x^2$ contains 4 in both its terms, we are able to alter its form, and write 4 with a^2-x^2 in a bracket, for if we remove the bracket the two expressions are evidently the same.

Ex. 5. $6(x^2-2y^2)-2x\{x+4(2y-3)\}+4\{x^2-2y(x-3y)-5y^2-6x\}$

$=6x^2-12y^2-2x\{x+8y-12\}+4\{x^2-2xy+6y^2-5y^2-6x\}$

$=6x^2-12y^2-2x^2-16xy+24x+4x^2-8xy+24y^2-20y^2-24x$

$=8x^2-24xy-8y^2=8(x^2-3xy-y^2)$. Ans.

Exercise VIII.

Reduce to their simplest form

1. $3a-2b+(5a-6b)-(7a-2b)$.
2. $4x^2-7y^2-(6x^2+4y^2)+(4x^2-10y^2)$.
3. $5a+3b+2c-(2a-3b)+(2a+4b-11c)$.
4. $x^2+3xy-y^2+(4xy-5y^2)-(2x^2-xy+5y^2)$.
5. $12x^3-(4y^3+5x^3)+(2x^3-y^3-x^3)-(2y^3-x^3)$.
6. $2a^2-(3ab-2ac+b^2)-(c^2+11ac)+(7a^2+b^2)$.
7. $4a+(2a-1)-(7a+15)-(a-1)+(2a+15)$.
8. $(5x-2y-3z)-(3x+4y)-(2x-6y+3z)$.
9. $(4a-3b-2c)-(2a-7b+4d)+(3a-4b)-(5a-4d)$.
10. $11x^2+4y^2-(2xy-3y^2)+(2x^2-3xy)-(3x^2-5xy)$.

11. $7a-\{3a+(2a-1)\}$.
12. $8x-\{6x+2y-(12x-3y)\}$.
13. $5x-2y+(3x-7y)-\{6x-(2x+5y)\}$.

14. $2a-\{3b-(4a-b)\}+\{2a-6b-(b-a)\}$.

15. $3x^2-\{4xy-(7x^2-2xy)\}-\{y^2-(3x^2+2xy+4y^2)\}$.

16. $\{2x^3-(x^3-x+1)\}-(3x-4)+\{2x^3+(12x-1)+6x\}$.

17. $\{x^2-(3x^2-2xy)+4y^2\}-\{x+(x^2-4)\}-(4x^2-3x+1)$.

18. $7x^2+(2x^3-3x-9)+\{(x^2-5x)+7-(x^3+3x)\}+12$.

19. $-\{3a-(4a-5b)+7c\}+\{(2a-3c)-(3a-4b+2c)\}$.

20. $7a^2-\{3a^2-(2b^2-3c^2)+(4a^2-5c^2)\}$
$+\{b^2-c^2-(3b^2+c^2)-9b^2\}$.

21. $3x^2-[4x+\{5x^2-(2x^2+3x)\}]$.

22. $4x-[3x-\{2x-(2x-5)+4\}]$.

23. $12x^3-[7x^3-5x+\{4x-(2x-1)\}+12]$.

24. $3a-[12a+\{(5b-3c)+4b\}]-\{2a-(3b+2c)\}$.

25. $4x+[7y-\{2z+(5-x)\}]-[4x+\{3y-(2z-7)\}]$.

26. $2x^3-4x^2-[3x-\{5x-(2x^2+7)\}]-(2x^3-6x^2+2x)$.

27. $4a-2b-[3a-2b+\{2a-2c-(4a-b-c)\}]$.

28. $7x^2-[2x-\{2x^2-(3x-4)\}-3x]+\{5x-(3x-7)\}$.

29. $-(4a^2-b^2)+\{2a^2-(b^2-3c^2)+4ab\}$
$+[4a^2-\{3b^2-(2c^2-bc)\}-ab]$.

30. $5x^2-[x^2-\{2y^2-(xy-2z^2)\}+3y^2]$
$+[2x^2+\{5xy-(y^2-z^2)\}]$.

31. $4a-\{3a-(2a-\overline{6a+1})\}-(4a-\overline{5a-2})$.

32. $7a^2-\{5a^2-2a+(2a^2-\overline{7a-5})\}+(3a^2-\overline{4a-12})$.

33. $\{5a^2-\overline{2a-7}\}-\{3a^2-(2a+\overline{5a-6})\}-(2a^2-7)$.

34. $2x-[3y-2z+\{4x-(5y-\overline{3x-2z})\}+5z]$.

35. $a-[2b+\{3c-(3a-\overline{a-b+c})\}+(2b-3c)]$.

36. $3x^2-[4x-5-\{2x^2-(7x+2)-(4x^2-\overline{2x-7})\}]$.

37. $5a-(7b+4c)-[6a-\overline{3b+2c}+4c-\{2a-(b-\overline{2a-c})\}]$.

38. $8x^2-\overline{5xy-6y^2}-[2x^2-\{3xy-y^2+(3x^2-\overline{2xy+5y^2})\}]$.

39. $-x^2-\overline{2ax-3a^2}+[a^2-x^2-\{2ax-(a^2-\overline{7ax+5x^2})\}-2ax]$.

40. $11a^3-(2a^2-3a)-[\{7a-\overline{2-9a^3}+(12a^2-\overline{11a-5})\}-a^3]$.

41. $11x-7+2(4x-3)+7(x-2)$.

42. $5x^2-3-(8x-1)-2(4x^2+5x-3)$.

43. $4a-11(2a+b-3c)+7(3a-b)$.

44. $6(a^2-2a+5)-2(a^3+a^2+1)-(a^3-a^2+2a-3)$.

45. $4(x^2-3x+2)-3(2x^2+4)-5(7x^2-3x-12)$.

46. $5x^2-(3x-\overline{x^2-4})+2(4x^2-\overline{x-5})+15$.

47. $2a^2-a(3a+1)+2(a^2-5a+6)-3(a^2-\overline{2a+1})$.

48. $6x^2+4x(y-z)+3(xy-\overline{xz-y^2})-3(x^2-2y^2)$.

49. $4a(a-b)-3(a^2-b\,.\,\overline{2a+b})+2a(a-b)$.

50. $a(a^2-2a)+10a^2(a-3)-5(a^3-a\,.\,\overline{a-2})$.

51. $a(a+b-c)-3b(a-b-c)+2c(a+b+2c)$.

52. $x(3x+2y-z)-3y(x+y+4z)+6z(2x-y-z)$.

53. $7(x^2+xy+y^2)-12x(y-2z)+3y(x+2z)$.

54. $2\{x^2+3(x^2-yz)\}-5\{xy-2y^2\}+7\{y^2-y\,.\,\overline{y-z}\}$.

55. $3a^2-[2a^2-2\{a^2-(a-1)\}+2]$.

56. $3[4x-5-2(3x-4)+5\{2x-3-(2x-7\,.\,\overline{x-5})\}]$.

57. $3(a^2-b^2)-[2a^2-2\{b^2+ab+b(b-a)\}]$.

58. $x-[\{2x-\overline{x-3(2x+y)}\}-3(2x+y)]$.

59. $7(a^4-a\,.\,\overline{a-2})+2a[a^2-\overline{a-4}]-3\{a(a^2-a)-a(a^3-\overline{2-a})\}$.

60. $-4[x^3-x^2(y-z)]+3xy(x-\overline{y-z})-2y\{x^2-3(xy-y^2)\}$.

CHAPTER IX.

Simple Equations.

A **simple** equation consists of two Algebraical expressions containing the first power of the same letter (usually x): these expressions have the sign = between them, and from this it is required to find the numerical value of the unknown letter.

If the equation has no fractional terms, begin the work by placing on the left side of the sign =, all the terms involving x; and on the right side all the terms without x; taking care to change the sign of every term which is moved from one side to the other. Then collect all the terms on each side into one term, and if x has any coefficient, divide both sides of the equation by it.

Ex. 1. $3x+8+7x=4x+56.$

First, place the terms with and without x on opposite sides; changing signs where necessary: thus we have

$$3x+7x-4x=56-8.$$

Simplifying, we obtain

$$6x=48;$$

$$\therefore\ x=8. \quad \text{Ans.}$$

Ex. 2. $12x-7-x+11=5x+19-2x-3.$

$$12x-x-5x+2x=7-11+19-3,$$

$$8x=12,$$

$$x=1\tfrac{1}{2}. \quad \text{Ans.}$$

Ex. 3. $5(x-4)+2(x-3)-3(x-5)=13.$

$$5x-20+2x-6-3x+15=13,$$

$$5x+2x-3x=20+6-15+13,$$

$$4x=24,$$

$$x=6. \quad \text{Ans.}$$

If the equation contain fractions, they must be first removed by multiplying both sides of the equation by the L.C.D. of the fractions.

Ex. 4. $\frac{x}{5}+\frac{x}{2}-4=\frac{3x}{10}-\frac{x}{4}+2\frac{1}{2}$.

The L.C.D. is 20; hence the equation becomes

$$4x+10x-80=6x-5x+50,$$
$$4x+10x-6x+5x=50+80,$$
$$13x=130,$$
$$x=10. \quad \text{Ans.}$$

Ex. 5. $\frac{2x-1}{3}+\frac{3x-5}{2}-\frac{6x-9}{7}=\frac{x-3}{2}+4.$

The L.C.D. is 42; hence we have

$$28x-14+63x-105-36x+54=21x-63+168,$$
$$28x+63x-36x-21x=14+105-54-63+168,$$
$$34x=170,$$
$$x=5. \quad \text{Ans.}$$

Ex. 6. $\frac{1}{4}(x-3)-\frac{1}{5}(2x-9)+\frac{3}{10}(3x-1)=6.$

This is only another form of a question like the last; it may be written

$$\frac{x-3}{4}-\frac{2x-9}{5}+\frac{9x-3}{10}=6,$$
$$5x-15-8x+36+18x-6=120,$$
$$15x=105,$$
$$x=7. \quad \text{Ans.}$$

Exercise IX.

Solve the following equations :

1. $9x+15=8x+19$.
2. $28x+17=27x+50$.
3. $41+31x=89+30x$.
4. $12x-17=11x-4$.
5. $63x-80=-7+62x$.
6. $50-7x=100-8x$.
7. $2x-7=x-4$.
8. $13x-41=58+12x$.
9. $16-49x=17-50x$.
10. $1328+67x=66x+2000$.
11. $12x+7=9x+13$.
12. $51x+7=44x+14$.
13. $13x+11=9x+27$.
14. $14x-20=8x+4$.
15. $17x-50=2x-5$.
16. $13x-21=19-7x$.
17. $48-7x=100-20x$.
18. $23+13x=48+8x$.
19. $57+4x=x+60$.
20. $90-2x=135-17x$.
21. $15x-21=14x-37$.
22. $24x+36=3+23x$.
23. $15x-18=13x-24$.
24. $17x+40=20+12x$.
25. $3x+15=4x+20$.
26. $7x-8=20+8x$.
27. $14x+17=29+18x$.
28. $21-7x=2x+57$.
29. $34-9x=51+8x$.
30. $4x-15=10x+21$.
31. $15x-20=13x-11$.
32. $7x-15=4x+20$.
33. $30x+12=18+25x$.
34. $29x-11=19-11x$.
35. $15+7x=12-8x$.
36. $2x-9=5x-17$.
37. $11+15x=12x+11$.
38. $16x-17=12x-31$.
39. $13x-70=15x+19$.
40. $77x+50=44x+28$
41. $12x+4+15x=8+11x+28$.
42. $13x-12-11=15+6x$.
43. $15x+51-12x=19-2x+32$.
44. $21x-14+15x-7=47-11x+26$.

45. $42x-11x+100=121-13x+23.$

46. $3x-7+4-2x=13-11x.$

47. $7x-10=3x+4-12x.$

48. $56+29x-17-13x=7x+31+20.$

49. $14x+70-9x=13+4x+93-7x.$

50. $23x-41+17-2x=56-11x+24.$

51. $9x+20-3x+7x=25-x.$

52. $17x=12-5x+21+7x+2.$

53. $43x+17-15x-10x-35=0.$

54. $15x-7x+15-3x-10=0.$

55. $12x-36+15x-21x+4+x=0.$

56. $14+9x-x+13+7=0.$

57. $97+13x-17+15x=x-10.$

58. $x=15x-60+14-11x+12.$

59. $64x-11x+27-13x=19x.$

60. $15-67-18x+12-21=2.$

61. $5\,(x-7)+2\,(x-9)=17.$

62. $7\,(x-4)+3\,(2x-1)=34.$

63. $3\,(3x-1)+10\,(x-3)=7x+15.$

64. $x-4-3\,(2x-9)=2\,(x-3)-41.$

65. $12(2x-1)-3\,(4x-3)-7\,(x-1)=9.$

66. $3\,(2x-1)-4\,(6x-5)=12\,(4x-5)-22.$

67. $4\,(x-3)-5\,(2x-3)=12-(x+9).$

68. $7\,(2x-3)-2\,(x-2)=3x-2\,(5x-9)+3.$

69. $10\,(2x-9)-7\,(4x-19)+5=4x-3\,(2x-3).$

70. $5x+3\,(7x-4)-2\,(10x-7)=4-(x-5).$

71. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=13.$

72. $\frac{x}{8}+\frac{x}{3}+\frac{x}{6}=15.$

73. $\frac{x}{10}+\frac{x}{5}=13+\frac{x}{25}$.

74. $\frac{x}{3}+\frac{x}{4}+\frac{x}{2}=x+8$.

75. $\frac{x}{2}-\frac{x}{5}+\frac{x}{4}=x-9$.

76. $\frac{x}{8}-\frac{x}{4}+\frac{x}{3}=\frac{x}{2}-7$.

77. $\frac{3x}{4}+x=\frac{7x}{8}+2x-9$.

78. $\frac{5x}{7}+3x=54+\frac{2x}{3}$.

79. $\frac{2x}{3}-\frac{5x}{6}=x-\frac{x}{2}+\frac{1}{3}$.

80. $-\frac{7x}{10}+1\frac{1}{2}+\frac{3x}{4}=2\frac{5}{6}-\frac{8x}{15}-1\frac{1}{3}$.

81. $\frac{5x-8}{4}+\frac{x-2}{5}=15$.

82. $\frac{8x+5}{12}=7+\frac{3x-4}{20}$.

83. $\frac{3x-5}{6}+\frac{4x-7}{8}=2\frac{1}{3}$.

84. $\frac{2x+7}{3}-\frac{x-5}{2}=6$.

85. $\frac{6x+7}{5}-\frac{2x-1}{10}=4\frac{1}{2}$.

86. $\frac{8x-9}{14}-\frac{3x-5}{21}+\frac{2x-7}{3}=0$.

87. $\frac{7x+2}{12}-\frac{4x+3}{15}=3\frac{4}{5}$.

88. $\frac{1}{8}(5x-10)-\frac{1}{12}(12x-13)=4x+\frac{1}{6}(2x-7)$.

89. $\frac{2}{3}(4x-1)-\frac{1}{7}(3x+2)=6+\frac{1}{9}(5x-2)$.

90. $\frac{1}{2}(4x-11)+\frac{1}{4}(3x-4)=3\frac{1}{3}-\frac{1}{12}(3x+13)$.

91. $\frac{2x+3}{11}-\frac{3x-7}{5}+x=6$.

92. $\frac{2x-5}{9}-\frac{x+3}{5}-\frac{x}{3}=2x-17\frac{1}{5}$.

93. $\frac{5x-7}{3}+\frac{2x-11}{6}-\frac{x}{2}=3x-4\frac{1}{6}$.

94. $3x-4+\frac{3x+5}{4}-\frac{7-4x}{3}=0$.

95. $\frac{x-2}{3}-\frac{2x-7}{14}=\frac{x}{2}+2$.

96. $\frac{5x-8}{2}-\frac{10x-7}{3}+2=\frac{5x}{4}-1\frac{1}{2}$.

97. $\frac{1}{5}(2x+11)-\frac{1}{2}(5-6x)=7x+1\frac{1}{2}$.

98. $2x-\frac{1}{9}(5x-2)+\frac{1}{3}(10x+17)+9\frac{2}{3}=0$.

99. $x+\frac{2}{5}(4x+3)+\frac{3}{4}(2x+1)=\frac{6x}{5}-11\frac{1}{10}$.

100. $3+4x-\frac{2}{7}(3x+4)=\frac{3}{5}(2x-5)$.

101. $\frac{4}{7}(2x-21)-\frac{2}{9}(4x+3)+4\frac{1}{3}=\frac{1}{3}(2x-25)$.

102. $\frac{1}{8}(3x+5)-\frac{2}{7}(11x+3)=\frac{3}{4}(x-5)$.

103. $9x-\frac{2}{5}(2x+1)-\frac{5}{6}(4x+7)=3\frac{1}{2}$.

104. $\frac{4x-1}{3}+\frac{2}{7}(x-5)=\frac{3x}{2}-2$.

105. $4(x-6)-2\{3x-(x-8)\}=5(13-3x)$.

106. $2(3x+4)=5\{2x-3(x+4)+9\}-(1-3x)$.

107. $8\{2(x-1)-(x+3)\}=5[x+7\{x-2(4-x)\}]$.

108. $4x-3\left\{5x-8\left(x+\frac{1}{2}\right)\right\}=13x+1$.

109. $4(2x+7)-3(3x+10)=\frac{x+5}{2}$.

110. $7(3x-5)-6(2x-3)=\frac{3x-1}{5}$.

111. $6(2x+3)-3(4x+5)=\frac{1}{3}-2x$.

112. $12\left(x+\frac{1}{3}\right)-(2x+5)=15x-\frac{3(x+3)}{2}$.

113. $3(2x-3)-\frac{x+7}{2}+\frac{3x-1}{5}=\frac{x-4}{4}$.

114. $2(x-1)+\frac{3x-5}{4}-\frac{4x-3}{9}+\frac{x-11}{2}=0$.

115. $\frac{1}{5}(x-1)+\frac{1}{7}(2x-3)-3=2(x+5)$.

116. $\frac{3}{4}(5x+1)-\frac{2}{9}(7x-2)-\frac{2x}{3}=\frac{1}{6}(x-1)$.

117. $2(3x+1)-\frac{2}{5}(x+4)+20=\frac{2}{3}(x+7)$.

118. $5x+\frac{3x-4}{6}-\frac{2}{5}(6x-1)-2(2+x)=0$.

119. $4(x+6)-\frac{x}{3}-\frac{2}{5}(x+10)=\frac{13}{10}(x+5)-1\frac{1}{4}$.

120. $5\{2x+1-3(x+1)\}-\frac{x+2}{3}+\frac{10}{7}(3x+5\frac{1}{2})+2\frac{3}{8}=0$.

CHAPTER X.

PROBLEMS.

IN a problem, we have given the relation which an unknown quantity bears to some given quantity, and from this we are required to find the unknown quantity: for example, we may be told to find a number such that if we double it and then add 24 to it, the result will be 100; from this we could determine the number.

When only one unknown quantity is to be found, we commence by saying that x is to represent it, and then write x for the unknown quantity in whatever is said of it in the problem: thus, in the example we have just given, taking x to be the required number we double it, which gives $2x$, and then add 24, which makes $2x+24$; this, we are told, is equal to 100; so that we obtain the equation

$$2x+24=100.$$

When the equation is formed, the value of x can be found by the ordinary method; thus in the example we have taken, we have

$$2x=100-24$$
$$=76;$$
$$\therefore x=38, \text{ the required number.}$$

Ex. 1. If I add 45*s.* to the money in my purse, I should have four times as much as I have at present; what is the sum in my purse?

Let x be the number of shillings I have now; then $x+45$ is what I shall have after adding 45*s.*, but this is to be four

times what I have at present;

$$\therefore\ x+45=4x.$$
$$x-4x=-45,$$
$$-3x=-45,$$
$$x=15.$$

15*s.* is the required sum. Ans.

Ex. 2. What is the number whose fourth and seventh parts added together amount to 176?

Let x be the required number: then $\frac{x}{4}$ is its fourth part, and $\frac{x}{7}$ is its seventh part; hence

$$\frac{x}{4}+\frac{x}{7}=176,$$
$$7x+4x=4928,$$
$$11x=4928,$$
$$x=448. \quad \text{Ans.}$$

Ex. 3. Two persons agreed to give £50 to a charity, one contributing £12 more than the other: what did each give?

Let x be the smaller contribution in £s., then $x+12$ is the larger; now the two together amount to £50; hence

$$x+x+12=50,$$
$$2x=50-12=38,$$
$$x=19.$$

£19 and £31 were the two contributions.

Ex. 4. The sixth part of a certain number exceeds its tenth part by 28; find the number.

Let x be the required number; its sixth part is equal to its tenth part increased by 28;

$$\therefore \frac{x}{6} = \frac{x}{10} + 28,$$

$$5x = 3x + 840,$$

$$2x = 840,$$

$$x = 420. \quad \text{Ans.}$$

EXERCISE X.

1. What number is that to which if you add 12, the sum is 4 times the original number?

2. What number is that from the double of which if you take 64, the remainder is 16?

3. A man had in his purse a certain sum of money; on adding 36*s.* to it, he found that his money was three times as much as he had at first; what sum had he?

4. A library contains 2500 books; what number must be added to it that the whole number may be eleven times the number added?

5. Find a number such that its third and fifth parts when added together amount to 64.

6. Find a number such that the sum of its fourth and tenth parts is 35.

7. Find a number such that the difference between its third and seventh parts is 44.

8. Find a number to the tenth part of which, if 4 be added, the sum is the fifth part of the original number.

9. A father's age is three times that of his son, and their ages added together amount to 48; find the son's age

10. A carriage horse cost twice as much as the carriage, and the price of the two together was £84; find what each cost.

11. The distance between two places A and B is four times the distance between B and C: if the difference of these distances be 51 miles, find the number of miles from A to B.

12. The difference of the number of boys in two schools is 450, one school having three times as many as the other; what are the numbers?

13. A and B together contributed £100 to a charity, A giving £15 more than B: find what each contributed.

14. Two boys have £2. 10*s.* between them; one has 8*s.* more than the other: how much has each boy?

15. Two engines are together of 80 horse power; one of the two is 16 horse power more than the other; find the power of each.

16. A, B, and C together gave £150 to a charity: B gave £10 more than A, and C gave £25 more than B: find the contribution of each.

17. A, B, and C have £1000 divided amongst them; A has £100 more than B, and B £50 more than C: what is the share of A?

18. A father left his three sons £20,000, the eldest to have £2000 more than either of the others: find the share of each.

19. Find a number such that the difference between its fourth and fifth parts is 7.

20. What number is that whose fifth part exceeds its twelfth part by 28?

21. What number is that to which if 16 be added, four times the sum will exceed ten times the number by 22?

22. Divide 40 into two parts, so that one part may be $\frac{2}{3}$ of the other.

23. Divide 60 into two parts, so that a third of one part may equal half the other.

24. Divide a guinea into two parts so that the value of one part may be $\frac{3}{4}$ that of the other.

25. Divide 81 into two parts such that one-thirteenth of the greater may be equal to twice the less.

26. Divide 192 into two parts, such that half of the greater shall exceed a third of the less by 21.

27. Find a number such that if 36 be taken from its double the remainder may be $\frac{4}{5}$ of the original number.

28. A father's age is 42, and his son's is 6: in how many years will the father be 3 times as old as his son?

29. *A* is 72, and *B* is two-thirds of that age; how long is it since *A* was 5 times as old as *B*?

30. *A* is 70 years old, *B* is exactly half that age: how many years have passed since *A* was $3\frac{1}{2}$ times the age of *B*?

31. *A* is 56 years old, *B* is $\frac{3}{4}$ of that age: how long is it since *B*'s age was $\frac{3}{5}$ of what *A*'s was?

32. *A* has £3 and *B* £2. 8*s*.: how much will *A* have to give *B* that *B* may have three times as much as *A*?

33. A father is four times as old as his son, twelve years ago he was sixteen times as old: find their present ages.

34. A had five times as much money as B; he paid B £1. 3*s.* and then had only twice as much: how much had A at first?

35. A has £20 and B £15: how much must B give A so that A may have 3 times as much money as B?

36. Divide £300 among A, B, and C, so that A shall have twice as much as B, and B £20 more than C.

37. Divide £760 among A, B, and C, so that B may have £50 more than A, and £135 less than C.

38. A spends a sum of money; B spends half of what A spends; C spends $\frac{3}{4}$ of what A and B together spend. If A, B, and C altogether spend £52. 10*s.*, what does each spend?

39. A, B, and C have a certain sum of money divided among them: A has £5 less than half; B has £3 less than A; C has £7. 10*s.* less than B: find the sum divided.

40. The difference of two numbers is 60; and one-seventh of one of them is equal to one-third of the other. Find them.

41. Find a number such that its half is as much above 100 as its third part is below 100.

42. Find a number which, after being multiplied by 4, will exceed 50 as much as it is now below 50.

43. The difference between two numbers is 20, and one-seventh of the greater exceeds one-tenth of the less by 5: find the numbers.

44. The difference of two numbers is 10, and half of one of them is greater than twice the other by 2: find them.

45. Find two numbers differing by 8, such that four times the less may exceed twice the greater by 10.

46. Divide 144 into two parts, such that the greater increased by 16 shall be double of the less diminished by 10.

47. Divide 70 into two parts such that the sum of the quotients obtained by dividing one part by 2 and the other by 3, may be equal to 31.

48. A and B have £77 between them; A and C £105; B and C £112: how much has each?

49. If A and B together have £49. 5*s.*; A and C together £38. 10*s.*; B and C together £52. 15*s.*: how much have they separately?

50. The sum of the sixth and eighth parts of a certain number exceeds its twelfth part by 20; find the number.

51. There is a number to which if 20 be added, one-sixth of the new number will equal a quarter of the original number; find the number.

52. A first class railway fare exceeds the second class for the same distance by 5*s.*, and a third of the first class is equal to half of the second class fare; find the fares.

53. What number must be taken away from 80, so that a sixth part of the number subtracted may equal half of the number remaining?

54. A house has 3 rows of windows, six in each row; the lowest row has 2 panes of glass in each window more than the middle row, and the middle row has 4 panes in each window more than the upper row; there are in all 168 panes of glass. How many are there in each window?

55. In ascending a mountain, a man took half as long again to climb the second third as he did to climb the first third, and a quarter as long again for the last third as for the second third; he took altogether 5 hrs. 50 min. Find the time he spent on the first part.

56. *A*'s annual income is £450 less than *B*'s: *A* lays by half of his, and thereby saves in two years £200 more than *B*, who only lays by one-sixth of his income: find their incomes.

57. Of a certain sum of money I spent a third part and put in its stead £50; then of the sum thus increased I spent one-fourth, and put in its stead £70; I then had £120: what was the original sum?

58. A sum of £7. 10*s*. is divided amongst a certain number of persons; if there had been 5 persons more they would each have received 5*s*. less: how many were there?

59. *A* obtains in shooting 3 more points than $\frac{2}{3}$ of *B*'s score; if he had obtained one less he would have had half *B*'s: by how many points was he beaten?

60. A man buys a case of oranges at 6*d*. a dozen: he finds fifty spoiled, and selling the rest at 3 for 2*d*. makes 10*s*. 9*d*. profit: how many are there in the case?

CHAPTER XI.

SIMULTANEOUS EQUATIONS WITH TWO UNKNOWN QUANTITIES.

In Simultaneous Equations we have more than one unknown letter, and as many equations as there are unknown letters.

When there are two unknown letters (x and y), multiply each of the equations by a number such that the coefficients of one of the letters become the same in the two equations: then subtract one from the other, or if these coefficients have different signs, add the equations, so that this letter will disappear, and the numerical value of the other letter

will immediately be found. To find the value of the remaining letter, substitute in either of the equations the value already found for one letter.

Ex. 1. Solve $3x+11y=64$; $4x+3y=27$.

Multiplying the first equation by 4, and the second by 3, we have

$$12x+44y=256$$
$$12x+\ 9y=\ 81;$$

whence we obtain by subtraction,

$$35y=175;$$
$$\therefore\quad y=\ 5.$$

To obtain x, substitute the value of y in the second equation, and we find

$$4x+15=27;$$
$$\therefore\ 4x=27-15=12;$$
$$\therefore\quad x=3.$$
$$x=3;\ y=5.\quad \text{Ans.}$$

When the equations involve fractions, each equation must be simplified by itself, so as to have on the left side one term involving x and one term involving y, and on the right hand a plain figure.

Ex. 2. $\dfrac{2x-5}{3}-\dfrac{x-y}{2}=4$; $\dfrac{y}{2}-\dfrac{2x-y}{5}=3\frac{1}{2}$.

The first equation becomes

$$4x-10-3x+3y=24$$
$$x+3y=34 \quad\ldots\ldots\ldots\ldots\ldots\ldots(1);$$

the second equation becomes

$$5y-4x+2y=35$$
$$-4x+7y=35 \quad\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

Multiply equation (1) by 4,

$$4x+12y=136,$$

and add to it equation (2) $-4x+7y=35$,

whence we have $19y=171$;

$$\therefore\ y=9.$$

Substituting this in equation (1), we find $x+27=34$;

$$\therefore\ x=7.$$

$$\left.\begin{matrix}x=7\\y=9\end{matrix}\right\}\text{ Ans.}$$

Ex. 3. $4x-\frac{1}{3}(2x-y)-\frac{1}{2}(6x-y)=0$; $\frac{2}{5}(4x+5y)=x-\frac{1}{2}$.

The first equation becomes

$$24x-4x+2y-18x+3y=0$$

$$2x+5y=0.....(1);$$

the second equation becomes

$$16x+20y=10x-5$$

$$6x+20y=-5 .....(2).$$

Multiply (1) by 3, $6x+15y=0$;

$$\therefore\ 5y=-5$$

$$y=-1.$$

Substitute in (1), $2x-5=0$

$$2x=5 \text{ or } x=2\tfrac{1}{2}.$$

$2\frac{1}{2}$; -1. Ans.

Exercise XI.

Solve the following equations:

1. $3x+4y=26$; $x+3y=17$.
2. $x+2y=19$; $4x+5y=55$.

3. $2x-5y=9$; $x+y=15$.
4. $x+9y=21$; $3x+5y=19$.
5. $3x+2y=46$; $4x+y=48$.
6. $2x-y=16$; $3x-7y=35$.
7. $3x+5y=37$; $2x+3y=23$.
8. $4x+5y=43$; $3x+2y=20$.
9. $7x+2y=71$; $2x+7y=1$.
10. $4x-5y=0$; $7x+6y=59$.
11. $5x-2y=25$; $8x-11y=1$.
12. $6x-3y=45$; $7x-5y=15$.
13. $10x+3y=82$; $21x+2y=155$.
14. $15x+4y=99$; $6x-5y=0$.
15. $12x+5y=13$; $15x+7y=20$.
16. $2x-3y=35$; $17x-4y=18$.
17. $2x-3y=71$; $9x+4y=22$.
18. $4x+7y=20$; $5x-11y=25$.
19. $5x-4y=28$; $8x+3y=-21$.
20. $5x+7y=0$; $12x-y=89$.
21. $6x+11y=7$; $4x=3-9y$.
22. $20x+21y=28$; $4y=15x+137$.
23. $18x=50+25y$; $5y+10=-27x$.
24. $56+10y=7x$; $15x=26+8y$.
25. $9y=2-11x$; $21x=27+6y$.
26. $3x-7y=25$; $4x=5y+29$.
27. $8x-59=3y$; $5y=7x-35$.
28. $25=5x+4y$; $3x-7y-15=0$.
29. $7y=80-6x$; $4x=66-11y$.
30. $13x+10y-103=0$; $9x=y$.

31. $3x-y=14=2x+4y.$

32. $5x+3y=11=y-2x.$

33. $2x-16=5y=3x-24.$

34. $7y=8x-5=10x-1.$

35. $\frac{x}{4}+\frac{y}{2}=7\,;\ \frac{x}{8}+\frac{y}{3}=4.$

36. $\frac{x}{5}-\frac{y}{10}=1\,;\ \frac{2x}{3}-\frac{y}{4}=5.$

37. $\frac{2x+y}{9}-\frac{y}{4}=1\,;\ \frac{x-y}{3}+\frac{x}{7}=2.$

38. $\frac{x}{5}-\frac{1}{7}(x-2y)=0\,;\ \frac{1}{10}(3x+5y)-\frac{1}{8}(x+y)=1.$

39. $\frac{3x+7y}{4}-\frac{x+y}{8}=5\,;\ 7x=56-3y.$

40. $\frac{1}{2}(x-2y)-\frac{1}{9}(2x-7)=1\,;\ y=x+17.$

41. $\frac{3x-y}{12}-\frac{7x-3y}{15}=\frac{1}{10}\,;\ \frac{x}{4}-\frac{y}{10}=\frac{3}{10}.$

42. $4x+\frac{x-y}{8}=21\,;\ 2y+\frac{3x-5y}{3}=4.$

43. $\frac{x}{4}+\frac{2x-y}{5}=2\tfrac{1}{2}\,;\ \frac{y}{7}+\frac{x-2y}{8}=0.$

44. $2x-\frac{5x-3y}{2}=\frac{y}{3}+1\,;\ 3y-\frac{7x-y}{4}+x=1\tfrac{1}{2}.$

45. $\frac{1}{3}(x-y)-1=\frac{1}{2}(2y-9x)\,;\ \frac{x}{4}-\frac{3y}{16}=1.$

46. $\frac{1}{5}(x-4y)-\frac{1}{4}(x+9y)=2\tfrac{1}{2}\,;\ \frac{1}{3}(2x+7y)=\frac{1}{2}(x+y).$

47. $\frac{4x-y}{2}=\frac{7x+3y}{11}+\frac{1}{2}\,;\ \frac{x}{4}+\frac{2x+3y}{8}=x.$

48. $\frac{5x}{7}-\frac{1}{10}(7x+4y)=2$; $\frac{1}{10}(x-2y)-\frac{1}{4}(2x+y)=2\frac{1}{4}$.

49. $\frac{5}{x}+\frac{12}{y}=7$; $\frac{1}{x}+\frac{6}{y}=2$.

50. $\frac{12}{x}-\frac{2}{y}=3$; $\frac{10}{x}-\frac{4}{y}=3\frac{2}{3}$.

51. $\frac{2}{x}-\frac{7}{y}=5$; $\frac{7}{x}-\frac{12}{y}=5$.

52. $\frac{16}{x}=3\frac{1}{3}+\frac{20}{y}$; $6y+5x=0$.

53. $\frac{x+3y}{3}-\frac{2x-7y}{8}=\frac{1}{3}=\frac{x}{12}-\frac{y}{4}$.

54. $3x-\frac{1}{4}(2x-y)=\frac{1}{6}-\frac{y}{3}=\frac{x-y-8}{4}-\frac{x+y}{10}+\frac{1}{5}$.

CHAPTER XII.

PROBLEMS PRODUCING SIMULTANEOUS EQUATIONS.

In some problems there are two unknown quantities, the connection between which cannot easily be expressed without assuming two letters to represent them. We therefore in such a case take x and y as the unknown quantities, and from what is given in the problem we shall be able to form two equations involving them: these equations are solved in the same manner as in the last exercise.

Ex. 1. Find two numbers such that the first increased by 12 is equal to the second diminished by 17, and that the sum of four times the first and seven times the second is 225.

Let x and y be the two numbers;

then $x+12=y-17$.....(1);

and $4x+7y=225$ ....(2).

Equation (1) becomes $x-y=-29$,

whence we have $4x-4y=-116$;

but by (2) $4x+7y=225$;

$\therefore\ -11y=-341$, and $y=31$.

Substituting in (1), $x-31=-29$, $\therefore\ x=2$.

2 and 31 are the required numbers. Ans.

Ex. 2. A and B have certain sums of money in their purses: if A were to give B 30s. their money would be the same, but if B were to give A 30s. A's money would then be twice B's.

Let x and y be the number of shillings A and B had respectively;

then, since by giving B 30s. A would diminish his own sum,

$$x-30=y+30 \quad \text{.....(1);}$$

also $$x+30=2(y-30) \quad \text{.....(2).}$$

From these we obtain $\left.\begin{matrix} x-y=60 \\ x-2y=-90 \end{matrix}\right\}$

hence $$y=150,\quad x=210.$$

A had £10. 10s.; B had £7. 10s. Ans.

Ex. 3. A grocer sold 20 lbs. of tea and 27 lbs. of coffee for £4. 15s.; he afterwards sold 16 lbs. of the same tea and 9 lbs. of the same coffee for £2. 15s.: find the price of the tea and coffee per lb.

Let x be the price of a lb. of tea, in shillings;

y ..... coffee .....

Then $$20x+27y=95 \quad \text{.....(1);}$$

$$16x+9y=55 \quad \text{.....(2).}$$

(2) becomes $48x+27y=165$,

but by (1) $20x+27y=95$;

$\therefore\ 28x=70$ or $x=2\frac{1}{2}$; whence $y=1\frac{2}{3}$.

The tea was 2s. 6d. a lb.; the coffee 1s. 8d. a lb. Ans.

Ex. 4. There is a number of two digits whose sum is 12; also, if the digits be transposed, the original number will exceed the new one by 18: find the number.

Let x and y be the digits;

then $$x+y=12 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

And since the original number is $10x+y$, and the new number is $10y+x$, we have

$$10x+y=10y+x+18 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2).$$

(2) becomes $$9x-9y=18,$$

$$\text{or} \quad x-y=2;$$

but by (1) $$x+y=12;$$

$\therefore$ $x=7$; $y=5$; and the required number is 75. Ans.

EXERCISE XII.

1. Find two numbers such that the sum of the first and twice the second is 31, and the sum of the second and twice the first is 26.

2. Find two numbers, such that the second together with twice the first is equal to 17, and twice the second together with the first is equal to 19.

3. Find two numbers such that twice the greater exceeds the less by 30, and five times the less exceeds the greater by 3.

4. A farmer sold to one person 9 horses and 7 cows for £300; and to another, at the same prices, 6 horses and 13 cows for the same sum: what was the price of each?

5. A man bought some sugar of two sorts, 7 lbs. of the first costing as much as 9 lbs. of the second, while 9 lbs. of the first and 7 lbs. of the second together cost 5*s.* 5*d.*; what was the price of each?

6. There are two numbers the half of the greater of which exceeds the less by 2, also a third of the greater exceeds half the less by 2: find the numbers.

7. A and B each bought a house, A's outlay exceeding B's by £400; if A's house had cost half of what it did, his outlay would have been £600 less than B's: what did each house cost?

8. Of the ages of two brothers one exceeds half the other by 4 years, and a fifth part of one brother's age is equal to an eighth of that of the other: find their ages.

9. If 3 years were added to the age of a father it would be twice that of his son, also if one year were taken from the son's age and added to the father's, the latter would still be twice the son's age: find their ages.

10. Four years ago a father was three times as old as his son is now, and the father's present age is twice what the son will be 8 years hence: what are their ages?

11. Six years ago a father was twelve times the age of his son, and 3 years hence the father will be three times as old as his son: find their ages at present.

12. A gentleman distributed £5 among 36 persons, giving 3*s*. each to some and 2*s*. 6*d*. each to the rest: how many were there of each class?

13. A farmer has oxen worth £12. 10*s*. each, and sheep worth £2. 5*s*. each, the number of oxen and sheep together being 35, and their value £191. 10*s*. Find the number he had of each.

14. The distance between two towns A and B exceeds by 12 miles the fifth part of the distance between B and C, and the whole length of the road from A to C through B is 24 miles: find the distance between A and B.

15. Find a number of two digits such that the sum of the digits is 10, and the number itself exceeds thirty times the difference of its digits by 4.

16. A number consists of two digits whose difference is 4: if the sum of the digits be multiplied by 4 the digits will be inverted: find the number.

17. There is a number of two digits which is equal to seven times the sum of the digits; also if the digits be transposed the new number will exceed 10 times the difference of the digits by 6: find the number.

18. A number of three digits whose first and last digits are the same, has 7 for the sum of its digits: if the number be increased by 90, the first and second digits will change places: find the number.

19. A man bought 5 sheep and 8 calves for £30; he sold four of the sheep at a gain of 10*s.* each and six of the calves at a gain of £1 each, for £31; what did he pay for each?

20. The perimeter of a rectangle is 100 feet: if its length be increased by 2 feet, and its breadth diminished by 2 feet, the area of the new rectangle will be 24 square feet less than that of the old one: find the length of the original rectangle.

21. A man buys 400 oranges, some at 3 for 2*d.* and the rest at 2 for 3*d.*: he sells them all at 1*d.* each, and neither gains nor loses: how many does he buy of each sort?

22. If a man's expenditure were £50 a year more than it is, he would spend three-fourths of his income; if he spent £15 a year less than he does, and his income were £225 more, his income would be double his expenditure. What is his income?

23. A ship was found at sea abandoned by four-fifths of its crew; it was reinforced by a part of the crew of the vessel that found it, so that each ship had two-thirds of its proper complement: the first vessel was then 10 hands short. What was the proper number of men for each vessel?

24. A certain number of half-guineas and half-crowns amount to £5. 12*s*. If there were 3 half-guineas more, and twice as many half-crowns, the amount would be £8. 18*s*. 6*d*. How many coins were there?

25. A besieging army add a certain number of feet to a trench during the day, and the besieged succeed in filling up a certain number of feet at night: the trench which has to be 945 yards long is finished in 21 days. If the besiegers were to set twice as many men to work at the trench, it would be finished in 9 days. What length do they dig in a day?

26. A can do a piece of work in 12 days; B and C together can do the same piece of work in 4 days; A and C can do it in half the time in which B alone can do it. How long will B and C take to do it separately?

27. A and B having equal sums engage in play: after some time it is found that if A had £6 more he would have 3 times what B has, and if he had £16 less he would have twice what B has. How much does A win? and what have they at first?

28. A boy bought a pint of strawberries: if he had given a farthing a piece, he would have got 6 more for his money; if he had given a penny more he would have bought them at the rate of 3 for a penny. How many did he buy, and what did he give?

29. An edition of a work is published consisting of a certain number of copies. It is observed that if the cost of the edition had been £100 more than two-thirds of what it was, the cost per volume would have been increased by 1*s.*; and if it had cost £100 more than three-fourths of its actual cost the price per volume would have been increased by 2*s.* Find the number of volumes and cost per volume.

30. A school is divided into a certain number of divisions. If there were 36 more boys in the school, the average number of boys in a division would be increased by three; and if the school were divided into 3 less divisions the number of boys in a division would be twice the number of divisions. How many boys are there in the school?

CHAPTER XIII.

GREATEST COMMON MEASURE.

WHEN two or more Algebraical expressions are arranged according to ascending or descending powers of some common letter, their G.C.M. is the highest expression with regard to that letter which will divide them without remainder.

The G.C.M. is found by the same method as in Arithmetic; you divide one expression by the other; then take the remainder for a new divisor, and the old divisor for a new dividend; continue this process until there is no remainder: the last divisor will be the G.C.M.

Ex. 1. Find the G. C. M. of $x^2+12x+35$ and $x^2+13x+42$.

$$
\begin{array}{l}
x^2+12x+35\,)\,x^2+13x+42\,(1\\
\qquad\underline{x^2+12x+35}\\
\qquad\quad x+7\,)\,x^2+12x+35\,(x+5\\
\qquad\qquad\underline{x^2+7x}\\
\qquad\qquad\quad 5x+35\\
\qquad\qquad\quad \underline{5x+35}
\end{array}
$$

$x+7$ is the G. C. M.

Ex. 2. Find the G. C. M. of

x^3+3x^2+4x+4 and x^3+4x^2+5x+6.

$$
\begin{array}{l}
x^3+3x^2+4x+4\,)\,x^3+4x^2+5x+6\,(1\\
\qquad\underline{x^3+3x^2+4x+4}\\
\qquad\quad x^2+x+2\,)\,x^3+3x^2+4x+4\,(x+2\\
\qquad\qquad\underline{x^3+x^2+2x}\\
\qquad\qquad\quad 2x^2+2x+4\\
\qquad\qquad\quad \underline{2x^2+2x+4}
\end{array}
$$

x^2+x+2 is the G. C. M.

If either of the expressions have any figure or letter common to all its terms, that figure or letter may be divided out of the whole expression, before we begin to find the G. C. M.; it being observed that if the same factor be divided out of *both* the expressions, it must be replaced in the answer, as a factor of the G. C. M.

In the same way, if during the process a remainder occur which has any letter or figure in all its terms, that letter or figure (not being common to both expressions and therefore not a part of the G. C. M.) may be removed before using the remainder as a new divisor.

Ex. 3. Find the G. C. M. of

$$2x^3+18x+36 \text{ and } 2x^3+20x^2+48x.$$

In this case, we can divide 2 out of the first expression, and $2x$ out of the second:

$$\begin{array}{l} x^2+9x+18\,)\,x^2+10x+24\,(1 \\ \qquad\quad x^2+\ 9x+18 \\ \qquad\quad \overline{\qquad\qquad} \\ \qquad\quad x+\ 6\,)\,x^2+9x+18\,(x+3 \\ \qquad\qquad\quad x^2+6x \\ \qquad\qquad\quad \overline{\qquad\qquad} \\ \qquad\qquad\qquad 3x+18 \\ \qquad\qquad\qquad 3x+18 \\ \qquad\qquad\qquad \overline{\qquad\qquad} \end{array}$$

Thus, $x+6$ is the G. C. M. of $x^2+9x+18$ and $x^2+10x+24$: and, since the factor 2 was taken out of both the given expressions, their G. C. M. is $2(x+6)$. Ans.

Ex. 4. Find the G. C. M. of

$$36x^4+30x^3-5x-1 \text{ and } 6x^3+17x^2+11x+2.$$

$$\begin{array}{l} 6x^3+17x^2+11x+2\,) \\ \quad 36x^4+\ 30x^3-\ \ 5x\ -1\,(6x-12 \\ \quad 36x^4+102x^3+\ 66x^2+12x \\ \quad \overline{\qquad\qquad\qquad\qquad} \\ \qquad -\ 72x^3-\ 66x^2-\ 17x-\ 1 \\ \qquad -\ 72x^3-204x^2-132x-24 \\ \qquad\quad \overline{\qquad\qquad\qquad\qquad} \\ \qquad\quad 23\ \big|\ 138x^2+115x+23 \\ \qquad\qquad\quad \overline{\qquad\qquad\qquad} \\ \qquad\qquad\quad 6x^2+\ 5x+\ 1\,)\,6x^3+17x^2+11x+2\,(x+2 \\ \qquad\qquad\qquad\qquad\quad 6x^3+5x^2+\ \ x \\ \qquad\qquad\qquad\qquad\quad \overline{\qquad\qquad\qquad} \\ \qquad\qquad\qquad\qquad\quad 12x^2+10x+2 \\ \qquad\qquad\qquad\qquad\quad 12x^2+10x+2 \\ \qquad\qquad\qquad\qquad\quad \overline{\qquad\qquad\qquad} \end{array}$$

$6x^2+5x+1$ is the G. C. M.

Ex. 5. Find the G. C. M. of

$$6x^3-5x^2-13x+12 \text{ and } 12x^3-8x^2-35x+6.$$

$6x^3-5x^2-13x+12$)
$12x^3-8x^2-35x+6$ (2
$12x^3-10x^2-26x+24$

$2x^2-9x-18$) $6x^3-5x^2-13x+12$ ($3x+11$
$6x^3-27x^2-54x$

$22x^2+41x+12$
$22x^2-99x-198$

$70 \mid 140x+210$

$2x+3$

$2x+3$) $2x^2-9x-18$ ($x-6$
$2x^2+3x$

$-12x-18$
$-12x-18$

$2x+3$ is the G.C.M.

It will sometimes be found that neither of the given expressions will divide into the other, or that a remainder occurs which will not divide into its dividend. In either of these cases we may multiply the dividend by such a quantity as will make the division possible.

Ex. 6. Find the G.C.M. of

$2x^3+3x^2-11x+3$ and $3x^4+9x^3-7x^2-12x+4$.

Multiplying the second expression by 2, we have

$2x^3+3x^2-11x+3$) $6x^4+18x^3-14x^2-24x+8$ ($3x$
$6x^4+9x^3-33x^2+9x$

$9x^3+19x^2-33x+8$

Again multiply by 2. 2

$18x^3+38x^2-66x+16$ (9
$18x^3+27x^3-99x+27$

$11 \mid 11x^2+33x-11$

x^2+3x-1

$$x^2+3x-1\,)\,2x^3+3x^2-11x+3\,(\,2x-3$$
$$2x^3+6x^2-\;2x$$
$$-3x^2-\;9x+3$$
$$-3x^2-\;9x+3$$

x^2+3x-1 is the G.C.M.

EXERCISE XIII.

Find the Greatest Common Measure of

1. x^2+4x+3 and x^2+5x+4.
2. x^2+2x+1 and $2x^2+5x+3$.
3. $3x^2+10x+8$ and $6x^2+23x+20$.
4. $5x^2+7x+2$ and $15x^2+26x+8$.
5. $9x^2y^2+18xy+5$ and $18x^2y^2+39xy+15$.
6. $a^2-11a+10$ and $a^2-10a+9$.
7. $3a^2b^2-5ab+2$ and $3a^2b^2-4ab+1$.
8. $10x^2-23x+12$ and $30x^2-67x+33$.
9. $7x^2+16x-15$ and $28x^2+71x-65$.
10. $12x^2y^2-19xy-21$ and $48x^2y^2-73xy-91$.
11. $3x^2+5x-8$ and $6x^2+19x+8$.
12. $2x^3-13x-7$ and $2x^3-x^2+3x+2$.
13. $11x^2+87xy-8y^2$ and $22x^3-13x^2y-10xy^2+y^3$.
14. $12x^2+35x+22$ and $24x^2-14x-33$.
15. $2a^2+19a+35$ and $6a^2+17a+5$.
16. $8x^2+18ax-5a^2$ and $24x^2-10ax+a^2$.
17. $10x^2-37x-36$ and $20x^2-19x-28$.
18. $6x^3-5x^2-2x+1$ and $12x^3-16x^2-x+5$.
19. $2x^3-7x^2-29x-6$ and $6x^3+13x^2-2x-1$.

20. $3x^4-13x^3a+23x^2a^2-21xa^3$ and $7x^3-42x^2a+77xa^2-84a^3$.

21. $2x^3-11x^2+5x+25$ and $7x^3-25x^2-23x+20$.

22. $14x^3-19x^2-10x-1$ and $4x^4-12x^3+9x^2-1$.

23. $6x^3-2x^2-21x+7$ and $8x^3-6x^2-28x+21$.

24. $9x^3+33x^2-21x-77$ and $3x^3+8x^2-8x+11$.

25. $7x^3-22x^2+31x-4$ and $21x^3-3x^2-7x+1$.

26. $2x^3-3x^2-19x-36$ and $6x^3-41x^2+67x-18$.

27. $6x^3-29x^2+31x-4$ and $9x^3-12x^2-15x+20$.

28. $3x^3-x^2-x-4$ and $6x^3+13x^2-31x+4$.

29. $22x^3-15x^2-9x+2$ and $33x^3-61x^2+54x-8$.

30. $14x^3-7x^2a-4xa^2+2a^3$ and $2x^3-15x^2a+9xa^2-a^3$.

31. $6x^2-41x+63$ and $12x^3-34x^2+17x-7$.

32. $12x^4-31x^3+14x^2$ and $36x^3-45x^2+74x-35$.

33. $5x^6-x^4+11x^3+12$ and $10x^6-7x^4-2x^3+8$.

34. $12x^3+2x^2-11x+3$ and $9x^3+2x-1$.

35. $9x^3-30x^2y+51xy^2-36y^3$ and $2x^3y-11x^2y^2+20xy^3-21y^4$.

36. $8x^4-22x^3-28x^2+77x$ and $(36x^3-12x^2-126x+42)xy$.

37. $6(x^4+2x^3-3x^2-4x+4)$ and $8(x^3+2x^2-x-2)$.

38. $2x^2(x^4-2x^3-7x^2+16x+7)$ and $6x(x^3-6x^2-26x-9)$.

39. $(3x^3+10x^2+x-14)x$ and $(6x^3-19x^2-32x+105)x^2$.

40. $x^5-6x^4+9x^3-4x$ and $x^2(4x^3-10x^2-14x-4)$.

CHAPTER XIV.

REDUCTION OF FRACTIONS.

FRACTIONS are reduced to lowest terms in Algebra in the same manner as in Arithmetic, viz. by dividing out of the numerator and denominator whatever factor is common to both. Thus in Arithmetic, $\frac{96}{132}=\frac{8}{11}$, because we can remove 12 from the numerator and denominator of the first fraction; and so in Algebra, $\frac{15a^2x^3y^3}{24a^5xy^5}=\frac{5x^2}{8a^3y^2}$, because we can divide both numerator and denominator by $3a^2xy^3$.

When the common factor of the numerator and denominator cannot be seen by inspection, it must be found by G.C.M., and removed from both by division.

Ex. 1. $\frac{56a^2x^3y^7}{64ax^5y^6}=\frac{7ay}{8x^2}$.

Ex. 2. $\frac{108a^4b^3c^{10}}{216b^3c^9x}=\frac{a^4c}{2bx}$.

Ex. 3. Reduce $\frac{x^2+3x+2}{2x^2+3x+1}$ to lowest terms.

$$
\begin{array}{l}
x^2+3x+2\,)\,2x^2+3x+1\,(\,2\\
\qquad\quad 2x^2+6x+4\\
\qquad -3\,\big|\,\overline{-3x-3}\\
\qquad\qquad x+1\,)\,x^2+3x+2\,(\,x+2\\
\qquad\qquad\quad\ x^2+\ x\\
\qquad\qquad\quad\ \overline{2x+2}\\
\qquad\qquad\quad\ 2x+2\\
\qquad\qquad\quad\ \overline{}
\end{array}
$$

$x+1$ is the G.C.M., and dividing numerator and denominator by it, we obtain $\frac{x+2}{2x+1}$. Ans.

Ex. 4. $\dfrac{4x^3+3x^2-20x-15}{5x^4+2x^3-25x^2-10x}$.

$4x^3+3x^2-20x-15\,)\,5x^3+2x^2-25x-10\,(\,4$

$20x^3+8x^2-100x-40\,(\,5$

$20x^3+15x^2-100x-75$

$-7\,\big|\,-7x^2+35$

$x^2-5\,)\,4x^3+3x^2-20x-15\,(\,4x+3$

$4x^3-20x$

$3x^2-15$

$3x^2-15$

x^2-5 is the G.C.M., and dividing numerator and denominator by it, we find $\dfrac{4x+3}{(5x+2)x}$. Ans.

Exercise XIV.

Reduce to their lowest terms:

1. $\dfrac{18a^7b^3}{30a^4b^5}$.
2. $\dfrac{15a^2b^5c^3}{24a^3b^4c^3}$.
3. $\dfrac{36x^6y^3z^5}{42xy^5z^3}$.
4. $\dfrac{100a^2b^5}{115a^2bc^4}$.
5. $\dfrac{27ax^2y^3}{54a^2x^2z^2}$.
6. $\dfrac{30a^3x^3z^5}{15ax^2y^7}$.
7. $\dfrac{108abx^2y^3}{120a^3x^3y}$.
8. $\dfrac{144x^3yz^2}{1728ab^2y^3}$.
9. $\dfrac{210p^3qx^4}{20p^5x^3y}$.
10. $\dfrac{12ab^3k^3l}{10a^3b^2lm}$.
11. $\dfrac{x^2-4x+3}{x^2-2x-3}$.
12. $\dfrac{x^2+5xy+6y^2}{x^2+xy-6y^2}$.
13. $\dfrac{x^2-8xy+15y^2}{x^2+3xy-40y^2}$.
14. $\dfrac{6a^2-7ab-3b^2}{6a^2+11ab+3b^2}$.
15. $\dfrac{x^2-6x+8}{4x^2-23x+28}$.
16. $\dfrac{x^2+4x-77}{x^2+18x+77}$.
17. $\dfrac{5x^2-7x-6}{5x^2-17x+14}$.
18. $\dfrac{p^2-5p+4}{p^2-7p+12}$.

19. $\dfrac{5x^4-2x^3-3}{5x^4-11x^3+6}$.

20. $\dfrac{x^3-10x+21}{x^3-11x+6}$.

21. $\dfrac{x^3-7x+6}{x^3-2x^2-8x-96}$.

22. $\dfrac{4x^3-5x+1}{12x^3-12x^2+4x-4}$.

23. $\dfrac{x^3+7x^2+17x+15}{x^3+8x^2+19x+12}$.

24. $\dfrac{x^3-4x^2+5}{x^3+1}$.

25. $\dfrac{x^3-4x^2+2x+3}{2x^4-9x^3+12x^2-7}$.

26. $\dfrac{x^3-ax^2-a^2x-2a^3}{x^4+a^2x^2+a^4}$.

27. $\dfrac{3x^3-12}{x^3-2x^2+5x-10}$.

28. $\dfrac{4x^3-4x-3}{6x^2-5x-6}$.

29. $\dfrac{12x^2+7xy+y^2}{28x^2+3xy-y^2}$.

30. $\dfrac{12x^2+x-1}{15x^2+8x+1}$.

31. $\dfrac{8x^3-6x-9}{6x^3-17x^2+12x}$.

32. $\dfrac{x^2+4x+4}{x^3+8}$.

33. $\dfrac{x^3+7x^2+16x+16}{2x^3+12x^2+22x+24}$.

34. $\dfrac{6x^3-13x+6}{9x^3-13x+6}$.

35. $\dfrac{14x^2-34x+12}{9x^2y-39xy+42y}$.

36. $\dfrac{3x^2+x-4}{2x^3-3x^2+1}$.

37. $\dfrac{2x^3-x^2-x+2}{6x^3-5x^2+4}$.

38. $\dfrac{x^5-2x^4+3x^3-3x^2+6x-9}{x^5+2x^4-3x^3-3x^2-6x+9}$.

39. $\dfrac{5ax^2-11ax+6a}{12x^3+x^2-13x}$.

40. $\dfrac{3p^3-2pq-q^3}{4p^3q-2p^2q^2-3pq^3+q^4}$.

41. $\dfrac{3x^4-4x^3-4x^2}{4x^3-11x^2+8x-4}$.

42. $\dfrac{6x^4+x^3-x}{4x^3-6x^2-4x+3}$.

43. $\dfrac{3x^4-6x^3+11x^2-4x+6}{4x^4-9x^3+14x^2-3x}$.

44. $\dfrac{2x^4+x^3-2x^2-4x-3}{2x^5-x^4-x^3-x^2-x-3}$.

45. $\dfrac{4x^4-6x^3y+18xy^3+8y^4}{10x^4+17x^3y-11xy^3-4y^4}$.

CHAPTER XV.

Least Common Multiple

(by G.C.M.).

When two or more Algebraical expressions are arranged according to ascending or descending powers of some common letter, their Least Common Multiple is the lowest expression with regard to that letter, which will contain the given expressions.

To obtain the L.C.M. of two expressions first find their G.C.M. and divide both of them by it: the L.C.M. is the product of these two quotients and the G.C.M.

The L.C.M. of three expressions is obtained, by first finding the L.C.M. of two of them, and then repeating the process for that result and the third expression.

Ex. 1. Find the L.C.M. of $6x^2+7x+2$ and $8x^2+18x+7$.

First, to find the G.C.M., multiply the expression by 3,

$$
\begin{array}{l}
6x^2+7x+2\,)\,24x^3+54x+21\,(\,4 \\
\qquad\quad 24x^3+28x+\ 8 \\
\qquad\quad \overline{\qquad\qquad 13\ \big|\ 26x+13} \\
\qquad\qquad\quad 2x+\ 1\,)\,6x^2+7x+2\,(\,3x+2 \\
\qquad\qquad\qquad\quad 6x^2+3x \\
\qquad\qquad\qquad\quad \overline{\qquad\qquad 4x+2} \\
\qquad\qquad\qquad\qquad 4x+2 \\
\qquad\qquad\qquad\quad \overline{\qquad\qquad\qquad}
\end{array}
$$

$2x+1$ is the G.C.M.; dividing the first expression by it we obtain $3x+2$, and dividing the second by it we obtain $4x+7$: hence the L.C.M. is $(2x+1)(3x+2)(4x+7)$.

Ex. 2. Find the L.C.M. of

$2x^3+5x^2+5x+3$, $4x^3+5x^2+5x+1$, and $16x^3+28x+6$.

We must first find the L.C.M. of the first two;

$$
\begin{array}{r|l}
2x^3+5x^2+5x+3 & 4x^3+5x^2+5x+1 \;(\;2 \\
 & \quad 4x^3+10x^2+10x+6 \\
 & -5\;\big|\;-5x^2-5x-5 \\
 & \qquad x^2+x+1 \;)\; 2x^3+5x^2+5x+3 \;(\;2x+3 \\
 & \qquad\qquad 2x^3+2x^2+2x \\
 & \qquad\qquad\qquad 3x^2+3x+3 \\
 & \qquad\qquad\qquad 3x^2+3x+3
\end{array}
$$

The G.C.M. is x^2+x+1, and the quotients, when it is divided into the two expressions, are $2x+3$ and $4x+1$: hence the L.C.M. is $(x^2+x+1)(2x+3)(4x+1)=8x^4+22x^3+25x^2+17x+3$.

Next, to find the L.C.M. of this result and the third expression: removing the factor 2 from $16x^3+28x+6$, we have

$$
\begin{array}{r|l}
8x^3+14x+3 & 8x^4+22x^3+25x^2+17x+3 \;(\;x^2+x+1 \\
 & \quad 8x^4+14x^3+3x^2 \\
 & \qquad 8x^3+22x^2+17x \\
 & \qquad 8x^3+14x^2+3x \\
 & \qquad\qquad 8x^2+14x+3 \\
 & \qquad\qquad 8x^2+14x+3
\end{array}
$$

Thus $8x^3+14x+3$ is the G.C.M., and the quotients are x^2+x+1 and 2; hence the L.C.M. of the three expressions is $2(x^2+x+1)(8x^3+14x+3)$. Ans.

EXERCISE XV.

Find the L.C.M. of

1. x^2-6x+8 and $x^2-8x+16$.
2. $x^2+5x-84$ and $x^2+21x+108$.
3. $3x^2-7x+4$ and $6x^2-10x+4$.

4. $2x^2-7x+5$ and $6x^2-23x+20$.

5. $2x^2+13x+15$ and $8x^2+10x-3$.

6. $x^2-18xy+32y^2$ and $x^2-9xy+14y^2$.

7. $2x^3+3x-20$ and $6x^3-25x^2-29x+10$.

8. x^4-8x^3+16 and $x^4-7x^3+28x-16$.

9. x^3-8x+7 and $2x^3-18x^2+46x-30$.

10. $x^3-15x+36$ and x^3-3x^2-2x+6.

11. $10x^2+49x-33$ and $10x^3+29x^2+14x-21$.

12. $x^3-x^2-7x+15$ and x^3+x^2-3x+9.

13. x^2+x-12 and x^3-5x^2+7x-3.

14. $x^2-3x-28$ and $x^3-5x^2-11x-21$.

15. $x^2-7x+10$ and $4x^3-25x^2+20x+25$.

16. $x^2-3x-70$ and $x^3-39x+70$.

17. $3x^3-13x^2+23x-21$ and $6x^3+x^2-44x+21$.

18. a^3-2a^2-2a-3 and a^3+2a^2+2a+1.

19. $6x^2-11x+5$ and $3x^3-2x^2-1$.

20. $4x^2-14x-30$ and $6x^3+7x^2-3x$.

21. $20x^4+x^2+1$ and $25x^4+5x^3-x-1$.

22. $4x^3-6x^2-4x+3$ and $6x^4+x^3-x$.

23. $3a^4-17a^3+49a^2-67a+8$ and $a^4-11a^3+35a^2-73a+24$.

24. x^3+4x+3 and x^5+5x^3+6.

25. $6x^3-10x^2-3x+5$ and $6x^3-7x^2-2x-5$.

26. $3x^3-15x+6$ and $2x^3-2x^2-4x$.

27. $3x^2-5x+2$ and $4x^3-4x^2-x+1$.

28. $2a^3-10a^2+12a$ and $3a^4-15a^3+24a^2-24$.

29. x^3-x^2+x+3 and $x^4+x^3-3x^2-x+2$.

30. $a^6-6a^4+9a^2-4$ and $a^6+a^5-2a^4+3a^3-a-2$.

31. x^2+6x+5, x^2+7x+6, $x^2+11x+30$.

32. $x^3+2x-15$, x^2-5x+6, x^3+5x^2+x+5.

33. x^2-4, $x^3-7x+10$, $x^3-5x^2+4x-20$.

34. $6x^3-7x-20$, $9x^2-16$, $6x^3-13x^2-9x+10$.

35. $2x^3-x^2+6x-3$, $2x^3-3x^2-x+1$,
$x^4-x^3+2x^2-3x-3$.

CHAPTER XVI.

SIMULTANEOUS EQUATIONS INVOLVING THREE UNKNOWN QUANTITIES.

WHEN the values of three unknown quantities are required, we generally have given three equations from which to find them. The method is similar to that for solving two simultaneous equations: we take together two of the equations, multiplying one or both so as to make one of the letters have the same coefficient; then by addition or subtraction this letter can be removed. Next, we take together one of the equations we have already used, and the remaining equation, and by a similar process eliminate from them the same letter. Thus, we shall have obtained two equations involving only two unknown quantities: they must be solved as in Exercise XI. When two of the three unknown letters are found, the third can at once be determined by substituting the values of the other two in any of the original equations.

Ex. 1. $3x+y+2z=11$, $4x+3y+5z=22$, $7x+4y+12z=38$.

In this case, we shall have smaller figures to deal with if we first eliminate y, because its coefficients are the least; multiplying then the first equation by 3 and combining it with the second, we have

$$\begin{array}{r} 9x+3y+6z=33 \\ 4x+3y+5z=22 \\ \hline 5x \quad + z=11 \end{array} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4).$$

Next, multiply the first equation by 4, and combine it with the third,

$$\begin{array}{r} 12x+4y+8z=44 \\ 7x+4y+12z=38 \\ \hline 5x \quad - 4z=6 \end{array} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5).$$

Subtracting (5) from (4) we have $5z=5$, or $z=1$.

Substituting in (4), $5x+1=11$, $\therefore x=2$.

Substituting in (1), $6+y+2=11$, $\therefore y=3$.

Hence the values are 2, 3, 1. Ans.

Ex. 2. $2x+3y=10$, $3x+2z=11$, $5y-4z=8$.

Here, since each of the equations contains only two letters, it will only be necessary to combine two of them, but care must be taken to *eliminate that letter which does not appear in the third equation.*

The first and second equations become

$$\begin{array}{r} 6x+9y=30 \\ 6x+4z=22 \\ \hline \therefore \; 9y-4z=8 \end{array}$$

but from (3) $\quad 5y-4z=8$

$$\therefore 4y=0 \text{ or } y=0.$$

Substituting in equation (1), $2x=10$, or $x=5$.

..... (3), $-4z=8$, or $z=-2$.

$\therefore$ 5, 0, -2 are the values. Ans.

Exercise XVI.

Solve the equations:

1. $x+2y+3z=20,\ 2x+3y+2z=18,\ 5x+2y+4z=29.$
2. $2x+5y-2z=4,\ 3x+7y+2z=35,\ 2x+2y+5z=40.$
3. $3x+y+7z=24,\ 2x+3y+4z=27,\ 5x+4y-16z=24.$
4. $4x+6y+3z=11,\ 3x-4y+z=19,\ 7x+5y+8z=44.$
5. $5x+2y-5z=23,\ 10x+3y+4z=0,\ 2x-5y-7z=1.$
6. $3x+9y-7z=7,\ 4x+3y-5z=5,\ 10x-y-z=1.$
7. $5x-3y+z=5,\ 2x+y+3z=19,\ x+2y-z=4.$
8. $3x+4y-5z=3,\ 4x-5y+3z=13,\ 5x+3y+4z=26.$
9. $3x+2y-7z=19,\ 2x-3y-5z=0,\ 4x+y+3z=8.$
10. $11x+y+z=16,\ 2x+3y+4z=20,\ 5x+6y+7z=38.$
11. $5x+3y-6z=4,\ 3x-y+2z=8,\ x-2y+2z=2.$
12. $4x-3y+2z=40,\ 5x+9y-7z=47,\ 9x+8y-3z=97.$
13. $3x+2y=8,\ 4x+3z=20,\ 3y+6z=27.$
14. $4x+7y=12,\ 5x-2z=5,\ 3y+4z=20.$
15. $2x+7y=15,\ 2y+3z=11,\ 4x+9z=23.$
16. $14x-3y=30,\ 9y-5z=34,\ 2x+y+z=33.$
17. $7x-2y=20,\ 2x-3z=4,\ 3y-4z=-9.$
18. $3x-5y=17,\ 2y-5z=10,\ x-2z=8.$
19. $2x-3y=8,\ 5y-9z=10,\ x+4y-2z=15.$
20. $2x+3z=27,\ x+4y+2z=9,\ 3x+3y+4z=31.$
21. $\frac{x}{3}+\frac{y}{4}+\frac{z}{3}=2,\ \frac{x}{6}+y+\frac{z}{10}=2,\ x+y+\frac{z}{4}=\frac{1}{4}.$
22. $\frac{x}{2}+y+\frac{z}{5}=1,\ \frac{x}{4}-\frac{y}{8}-\frac{z}{10}=2,\ 3x-4z=4.$
23. $\frac{x}{5}-\frac{y}{3}=0,\ \frac{x}{2}+\frac{z}{5}=5,\ x-y+z=4.$

24. $\frac{x}{4}=3-\frac{z}{3}$, $\frac{x}{10}=1-\frac{y}{8}$, $2=z-y$.

25. $y=1\frac{1}{2}-\frac{z}{2}$, $x=5y+z$, $z=\frac{x}{2}$.

26. $\frac{x}{5}+\frac{y}{3}=\frac{z}{4}$, $\frac{x}{10}-\frac{y}{12}=\frac{1}{4}$, $x+y=z$.

27. $x+\frac{z}{2}=0$, $\frac{x+z}{7}=\frac{y}{6}$, $\frac{x}{2}=\frac{y-z}{3}-\frac{5}{6}$.

28. $x+2z=4x+2y+1=5y+3z-2=13$.

29. $\frac{1}{2}(3x+y+z)=x+2y+z=\frac{2x-3y+2z-2}{3}=z+2$.

30. $\frac{x}{2}+2y+z=0$, $3y-2z=\frac{x}{8}=\frac{x-y}{3}+\frac{z}{2}-1$.

CHAPTER XVII.

Quadratic Equations.

A QUADRATIC equation involves the square of the unknown quantity (e.g. x^2), but no higher power.

If the quadratic equation, when simplified, contain no term involving the first power of the unknown quantity (x), it is said to be a *pure quadratic equation;* but if such a term be in it, it is called an *adfected quadratic equation;*

thus $5x^2=80$ is a pure quadratic equation;

$5x^2+6x=32$ is an adfected .....

A pure quadratic equation is solved by simplifying it until the value of the square of the unknown quantity is determined, and then taking the square root of both sides

of the equation. The student will observe that, since the product of two negative quantities is positive, (e.g. $-a \times -a = +a^2$), every square number has two roots, one positive, the other negative: thus

the square root of 16 is $+4$ or -4, which is written ± 4,
..... 49 is $+7$ or -7 ..... ± 7,
..... 144 is $+12$ or -12 ..... ± 12.

Ex. 1. Solve the equation $11x^2 = 99$.

Simplify by dividing by 11; then $x^2 = 9$;

$\therefore x = \pm 3$. Ans.

Ex. 2. Solve $\frac{x^2+x}{2} - \frac{x^2-x+16}{3} = \frac{5x-x^2}{6}$.

The equation becomes $3x^2+3x-2x^2+2x-32 = 5x-x^2$,

whence $2x^2 = 32$ or $x^2 = 16$;

$\therefore x = \pm 4$. Ans.

The solution of an adfected quadratic equation is effected by adding to both sides of the equation a quantity which will make the side involving the unknown letter a perfect square; we can readily determine the rule for this by observing in a few instances the connection between the terms of a square; thus

the square of $x+4$ is $x^2+8x+16$,
..... $x+7$ is $x^2+14x+49$,
..... $x-10$ is $x^2-20x+100$,
..... $x-2a$ is $x^2-4ax+4a^2$.

It will be seen from these examples that the last term of the square may always be found by taking half the coefficient of the second term and squaring it; for

the half of 8 is 4, and $4^2=16$;
..... 14 is 7, and $7^2=49$;
..... -20 is -10, and $(-10)^2=100$;
..... $-4a$ is $-2a$, and $(-2a)^2=4a^2$.

We have, therefore, the following rule for solving an affected quadratic equation, in which x is the unknown quantity: Simplify the equation, and arrange it so as to make x^2 the first term, and the term involving x the second term, and place the term without x on the other side of the equation; then complete the equation by *taking half the coefficient of the second term, and squaring it, and adding this square to both sides of the equation.* Now take the square root of both sides of the equation, and solve the equation thus obtained as in simple equations.

Ex. 3. Solve $x^2+12x=85$.

Half of 12 is 6, and $6^2=36$;

hence $$x^2+12x+36=85+36$$
$$=121.$$

Taking the square roots, $x+6=\pm 11$;

$$\therefore x=\pm 11-6$$
$$=5 \text{ or } -17. \quad \text{Ans.}$$

Ex. 4. Solve $x^2-2x=24$.

Half of -2 is -1, and $(-1)^2=+1$;

hence $$x^2-2x+1=24+1$$
$$=25;$$
$$\therefore x-1=\pm 5;$$
$$\therefore x=\pm 5+1$$
$$=6 \text{ or } -4. \quad \text{Ans.}$$

Ex. 5. Solve $x^2-\frac{x}{3}=3\frac{1}{4}$.

Half of $-\frac{1}{3}$ is $-\frac{1}{6}$, and $\left(-\frac{1}{6}\right)^2=\frac{1}{36}$;

hence $$x^2-\frac{x}{3}+\frac{1}{36}=3\tfrac{1}{3}+\frac{1}{36}$$

$$=\frac{121}{36};$$

$$\therefore\ x-\frac{1}{6}=\pm\frac{11}{6};$$

$$\therefore\ x=\pm\frac{11}{6}+\frac{1}{6}$$

$$=2 \text{ or } -1\tfrac{2}{3}. \quad \text{Ans.}$$

Ex. 6. Solve $5x-2x^2=2$.

Rearranging the equation, we have $-2x^2+5x=2$;

and, dividing by -2, $x^2-\frac{5x}{2}=-1$.

Half of $-\frac{5}{2}$ is $-\frac{5}{4}$, and $\left(-\frac{5}{4}\right)^2=\frac{25}{16}$.

Hence $$x^2-\frac{5x}{2}+\frac{25}{16}=-1+\frac{25}{16}$$

$$=\frac{9}{16};$$

$$\therefore\ x-\frac{5}{4}=\pm\frac{3}{4};$$

$$\therefore x=\frac{5}{4}\pm\frac{3}{4}=2 \text{ or } \frac{1}{2}. \quad \text{Ans.}$$

Exercise XVII.

Solve the following equations:

1. $x^2=81$.
2. $x^2=144$.
3. $7x^2=847$.
4. $12x^2=108$.
5. $2x^2-72=0$.
6. $5x^2-125=0$.
7. $\frac{x^2}{2}-\frac{x^2}{9}=14$.
8. $\frac{x^2-2x}{3}+\frac{2x^2-3x}{4}=\frac{10-5x}{12}-x$.

9. $\frac{5-2x}{4}-\frac{1-x+x^2}{2}+2\frac{1}{2}=0.$

10. $\frac{1}{3}(7x^2+5x)-\frac{1}{6}(x^2+x)=\frac{3x}{2}.$

11. $x^2+6x=16.$

12. $x^2+10x=39.$

13. $x^2+14x=15.$

14. $x^2+20x=96.$

15. $x^2+4x=32.$

16. $x^2+4x=140.$

17. $x^2+2x=15.$

18. $x^2+2x=63.$

19. $x^2+48x=580.$

20. $x^2+100x=2400.$

21. $x^2-12x=28.$

22. $x^2-6x=27.$

23. $x^2-2x=24.$

24. $x^2-2x=0.$

25. $x^2-6x=-5.$

26. $x^2-8x=-12.$

27. $x^2-20x=-36.$

28. $x^2-84x=-243.$

29. $x^2+96x=-95.$

30. $x^2+42x=-320.$

31. $x^2+7x=18.$

32. $x^2+3x=10.$

33. $x^2+5x=24.$

34. $x^2+9x=70.$

35. $x^2-11x=26.$

36. $x^2-21x=100.$

37. $x^2+x=20.$

38. $x^2+x=156.$

39. $x^2-13x=-22.$

40. $x^2-53x=-150.$

41. $3x^2+6x=24.$

42. $8x^2+24x=32.$

43. $10x^2+40x=50.$

44. $9x^2+189x=900.$

45. $12x^2-12x=24.$

46. $x^2+6x-16=0.$

47. $x^2+10x-75=0.$

48. $x^2+9x-22=0.$

49. $x^2-7x+12=0.$

50. $x^2-5x-66=0.$

51. $x^2+\frac{3x}{2}=7.$

52. $x^2+\frac{2}{3}x=87.$

53. $x^2+\frac{7x}{4}=23.$

54. $x^2+\frac{2x}{5}=51\frac{4}{5}.$

55. $x^2+\frac{3x}{10}=17\frac{1}{5}.$

56. $x^2-\frac{x}{4}=34\frac{1}{2}.$

57. $3x^2+x=14.$

58. $5x^2+3x=92.$

59. $12x^2+7x=62.$

60. $9x^2-6x=24.$

61. $-x^2+5x=6.$

62. $-x^2+3x=-4.$

63. $7x+x^2=30.$

64. $40+6x+2x^2=76.$

65. $2x+11+5x^2=315.$

66. $4x^2+7x=3x^2+3x+5.$

67. $7x^2+9=5x^2+2x+49.$

68. $8x^2=6x^2+3x+2.$

69. $5x+15=10x^2.$

70. $14x-11=2x^2+20x-7.$

71. $x^2+\frac{1}{2}=\frac{11}{2}(x+2).$

72. $2x^2+3x+5=3x^2+4x-1.$

73. $28-8x=3x^2.$

74. $\frac{x^2}{3}=9+\frac{x}{2}.$

75. $\frac{2}{3}(x^2-x)-\frac{1}{4}(2x^2-3)=\frac{x}{12}.$

76. $\frac{1}{5}(x-2)-\frac{1}{7}(x^2-6x)=x-7.$

77. $\frac{x^2+2}{3}-\frac{5}{12}(3x^2-5x)=x-1\frac{1}{3}.$

78. $\frac{1}{6}(4x^2-3)-\frac{1}{5}(x^2-2x)=x-\frac{1}{12}.$

79. $\frac{5}{4}(x^2-2x)+(3-10x)x=\frac{x}{7}(4-15x)-\frac{1}{4}.$

80. $\frac{9x^2-1}{3}-\frac{x}{4}(1+9x)=x^2-x-1\frac{7}{3}.$

Exercise XVIII.

Miscellaneous Examples.

1. If $a=2$, $b=1$, $c=0$, $d=3$, find the value of
$$abc+abd-acd+2ab^2+d^3.$$

2. Add together
$$7x-5y+4z,\ 2y+x-11z,\ 3x+5z-y,\ 5x-3y-z.$$

3. From $5a-3b+6c$ take $2a+5b-4c$.

4. Multiply $6a^3+5a^2b-4ab^2+3b^3$ by $a^2-2ab+b^2$.

5. Divide $x^4-3x^3+2x^2-x+1$ by $x-1$.

6. Divide $x^{10}-x$ by x^3-1.

7. Find the square root of
$$a^6-4a^5+10a^4-12a^3+9a^2.$$

8. Simplify $2a^3+3a^2-[7a^2-2(5a-a^3)-4(a^2-3a)]$.

9. Solve the equation $\frac{2x-1}{4}-\frac{4x-1}{3}=x-4\frac{1}{2}$.

10. Find a number such that if 32 be added to its double, the result will be ten times the original number.

11. If $a=0$, $b=1$, $c=2$, $d=4$, find the numerical value of $abc+3bcd+2b^2d+3cd^2$.

12. Add together $12a^3+5a^2+a-10$, $a^3+4a-12$, $6a^2+7a$, and $2a^2+a+9$.

13. From $4a^3-3a^2b+2ab^2+9$ take $a^3-a^2b-3ab^2-b^3$.

14. Multiply x^3+3x^2-3x-2 by x^3-2x+4.

15. Divide $6a^2+5ab-4b^2$ by $3a+4b$.

16. Divide x^6-y^6 by x^2+xy+y^2.

17. Extract the square root of
$$4-4x-11x^2+14x^3+5x^4-12x^5+4x^6.$$

18. Solve the equations:

(i) $\frac{x}{8}-\frac{2x-1}{5}=\frac{3x-4}{15}-\frac{x}{120}$;

(ii) $7x+4(5-x)=11x-12.$

19. Solve the equations:
$$\frac{x}{3}+\frac{y}{5}=8;\quad \frac{x}{9}-\frac{y}{10}=1.$$

20. Find the square root of 60902416.

21. If $a=4, b=6, c=8$, find the value of
$$12a-3b+bc-a^2;$$
also of
$$\frac{2a+3c}{2b}+\frac{c^2-b^2}{b+c}-\frac{a+b+c}{3}.$$

22. Add together $7a^3-2a^2b+3ab^2+7b^3$,
$11a^2b-2ab^2+2b^3$, $-10a^2b-10b^3$, and $11a^3-2a^2b-b^3$.

23. Subtract
$7x^3-3x^2y+5xy^2-6y^3$ from $9x^3-2x^2y+4y^3$.

24. Multiply $3a^3-2a^2x-4x^3$ by $2a^2-7ax+x^2$.

25. Divide $2x^3+3x^2-2x-3$ by x^2-1

26. Divide a^6+2a^3+1 by a^2+2a+1.

27. Extract the square root of
$100x^6-140x^5+129x^4-116x^3+58x^2-24x+9$.

28. Solve the equations:

(i) $x+\frac{1}{5}(x-7)-\frac{1}{7}(2x-3)=10;$

(ii) $\frac{2x-y}{2}+14=18;\quad \frac{2y+x}{3}+16=19.$

29. Find the number whose fifth part exceeds its eighth part by $7\frac{1}{2}$.

30. Solve

$$3x-y+z=17,\ 5x+3y-2z=10,\ 7x+4y-5z=3.$$

31. If $a=3$, $b=5$, $c=7$, $d=1$, $e=0$, find the value of

$$2a^3+3abe-4cd^2-5a^2e-b^2d.$$

32. Add together $3x^2+y^2-2yz-z^2$, $2xy-3y^2+3yz$, and $2z^2-4x^2-2xy+y^2$.

33. From $2a^3-3ab^2+2b^3$ subtract $a^3-2a^2b+ab^2+4b^3$.

34. Multiply $3a^2-4ab+2b^2$ by $2a^2-4ab-3b^2$.

35. Divide $6x^4-17x^3-22x^2+26x+7$ by $2x-7$.

36. Multiply $a^2+2ab+b^2-c^2$ by $a^2-2ab+b^2-c^2$, and divide the product by $a^2-b^2+2bc-c^2$.

37. Extract the square root of

$$16-8a+a^2+40a^3-10a^4+25a^6.$$

38. Solve the following equations:

(i) $\dfrac{7x+2}{10}-12-\dfrac{3x}{4}=\dfrac{3x+13}{5}-\dfrac{17x}{4}$;

(ii) $\left.\begin{matrix}17x-69y=-103\\14x-13y=-41\end{matrix}\right\}$.

39. Simplify

$$5x^2+2y^2-[3x^2-2\{x^2-3y^2+4(y^2-z^2)\}+x^2].$$

40. Reduce $\dfrac{6x^2+11x+3}{12x^3+18x^2-14x-21}$ to lowest terms.

41. If $a=3$, $b=4$, $c=5$, $d=0$, find the value of

(i) $4a^2+6b^2+3ab$;

(ii) $3a^2bc-(c^2-b^2)+3(abd+b^3)$.

42. Add together $3x^3-2xy^2+y^3$, $3x^2y-2xy^2+3y^3$, $2x^3-3x^2y$, and $2xy^2-4y^3$.

43. Simplify $12a-[4a+\{3a-2b-(4b-5a)\}]$.

44. Multiply $2x^2-3xy+5y^2$ by $2x^2+3xy-5y^2$.

45. Divide $6x^4-23x^3+13x^2+22x-8$ by $3x^2-7x+2$.

46. Divide a^6-b^6 by $a^3-2a^2b+2ab^2-b^3$.

47. From the sum of $2x^3+4a^2x-a^3$, $2ax^2+3a^3$, and $x^3-ax^2-ax^2$ take the difference between $x^3+3ax^2+a^3$ and x^3-a^3.

48. Simplify

$$4x^3-3x^2-\{2x^3-3(2x+5)+2(7x^2-\overline{5x-9})\}.$$

49. Solve the following equations:

(i) $\dfrac{9x+11}{2}-x-\dfrac{x-2}{7}=36$;

(ii) $2x-\dfrac{y+3}{4}=7+\dfrac{3y-2x}{5}$; $5y+\dfrac{x-2}{3}=26$.

50. A contribution of £129 was raised by A, B, and C. A gave £15 more than B, and B gave £21 more than C: what did A give?

51. If $a=2$, $b=5$, $c=8$, $d=0$, find the value of

$$4ab-7ac-ab^2+2c^2+cd.$$

52. Subtract

$7a^3-3ab+3ac-b^3$ from $11a^3-3ab-3ac+2b^3$.

53. Multiply $2a^3-2a^2b-4ab^2+b^3$ by $2a^2-ab+3b^2$.

54. Divide $1-6x^5+5x^6$ by $1-2x+x^2$.

55. Simplify

$$2[x^2-3(x-1)+5]-3[x^2-\{x-2(x^2-1)\}+2x].$$

56. Solve the equation $\frac{1}{3}(x-2)-\frac{1}{12}(2-x)=\frac{5}{2}$.

57. Solve the equations

$$\left.\begin{aligned} 3x-2y&=11\\ 2x+\ y&=12 \end{aligned}\right\}.$$

58. Find the square root of

$9a^4-12a^3b+22a^2b^2-12ab^3+9b^4$ and of 316743·84.

59. Find the G.C.M of $2x^2-x-36$ and $6x^2-29x+9$.

60. A father is twice as old as his son; 20 years ago he was four times as old. Find the son's age.

61. Add together $3a^2+4ab+5c^2-6bc-8b^2+2ac$, $3b^2-4ac-3ab+c^2-5a^2$, and $2a^2-6c^2+5b^2-ab+6bc+2ac$.

62. Multiply $x^2+2xy-3y^2$ by $x^2-5xy+4y^2$; and $a^4+a^3+a^2+a+1$ by $a-1$.

63. Divide

$12a^4-26a^3b-8a^2b^2+10ab^3-8b^4$ by $3a^2-2ab+b^2$.

64. Simplify

$\{2a^2-(3ab-b^2)\}-\{a^2-(4ab+b^2)\}+\{2b^2-(a^2-ab)\}$.

65. Solve the equations:

(i) $2x-\frac{5x-4}{6}=7-\frac{1-2x}{5}$;

(ii) $x-\frac{y-2}{7}=5$; $4y-\frac{x+10}{3}=3$.

66. Reduce $\frac{2x^3+x-28}{6x^3-21x^2-4x+14}$ to its lowest terms.

67. Find the L.C.M. of

$3x^3-5x^2-3x+5$ and $3x^3-8x^2-x+10$.

68. Solve

$x+2y+3z=17,\ 2x-3y+z=0,\ 3x+y-5z=-15.$

69. Divide 84 into two parts, such that twice the greater shall be equal to five times the less.

70. Find two numbers, such that the greater and twice the less amount to 24, and the less and twice the greater amount to 27.

71. Add together

$x^3-3ax^2-3a^2x-a^3,\ 4x^3-5ax^2+6a^2x-15a^3,$
$3x^3+4ax^2+2a^2x+6a^3,\ -17x^3+19ax^2-15a^2x+8a^3,$
$-13ax^2-27a^2x+18a^3.$

72. From $3x^2+2xy-y^2$ take the sum of $-x^2-3xy+3y^2$ and $3x^2+4xy-5y^2$.

73. Multiply a^2+2a-1 by a^2-a+1; and
$9a^2-3ab+b^2-6a-2b+4$ by $3a+b+2$.

74. Divide $6a^2-16ab+8b^2$ by $2a-4b$; and
$4x^4y^4+1$ by $2x^2y^2-2xy+1$.

75. Find the square root of
$x^6-4x^5y+8x^4y^2-10x^3y^3+8x^2y^4-4xy^5+y^6$.

76. Solve the equations

$$\left.\begin{aligned}7x-3y+8&=x-1\\3x+5y-10&=4y+3\end{aligned}\right\}.$$

77. Solve

$2x+3y+4z=1,\ 3x+4y+5z=2,\ 4x+5y+z=8.$

78. Find the square root of ·0002 to 5 places of decimals.

79. £500 was divided between A and B, so that one-fifth of A's share was equal to one-third of B's: how much did each receive?

80. The length of a room is double its breadth; if the breadth be increased by 9 ft., and the length diminished by 10 ft., the area will remain unchanged: find the length of the room.

81. If $a=5, b=3, c=1, d=0$, find the numerical value of

$$\frac{60ac^3}{b}+\frac{a^2b}{10}-\frac{abd}{2}-\frac{6a^2c^2-b^4}{4b-6d}.$$

82. Multiply $a^3-6a^2b-5ab^2$ by $3a^2-ab$.

83. Find the cube of x^2-5y.

84. Find the square root of $4a^4-12a^3+13a^2-6a+1$.

85. Simplify

$$a^2-3(4a^2-3ab+b^2)-6\{b^2-2(a^2-ab)-3ab\}.$$

86. Find the G.C.M. of $3x^2-5x+2$ and $3x^3-4x^2+1$; also find their L.C.M.

87. Solve the equations:

(i) $\frac{x+2}{5}=\frac{x-2}{2}-\frac{x-1}{7}$;

(ii) $5x+2y=16,\ 7x-3y=5$;

(iii) $-x+y+z=-5,\ -x-y+z=-25,\ x+y+z=35$.

88. Find the fourth root of 23·4256.

89. Reduce $\frac{3x^4-11x^2+10}{6x^4-3x^3-22x^2+5x+20}$ to its lowest terms.

90. Find the difference between

$$a^3+b^2+c^3 \text{ and } (a+b+c)^3,$$

when $a=1, b=2, c=4$.

91. Add together $3x^3+7x^2y-3xy^2+y^3$, $3x^2y-3y^3$, $4x^3-8xy^2+y^3$, and $7x^2y+y^3+11xy^2$.

92. Subtract $7a^3b^3-3a^2b^2+4ab-7$ from
$$9a^3b^3+a^2b^2-ab-2.$$

93. Multiply $7a^2-3ab+2b^2$ by $2a^2-4ab-3b^2$.

94. Divide $a^4+4a^3b+6a^2b^2+4ab^3+b^4$ by $a^2+2ab+b^2$.

95. Divide $64a^6-x^{12}$ by $2a-x^2$.

96. Simplify $x^3-[(x-y)^2-\{(x-y-z)^2-(z-x)^2\}]$ when $x=5$, $y=3$, $z=1$.

97. Find the square root of
$$a^6+4a^5+2a^4+9a^2-4a+4.$$

98. Solve the equations:

(i) $(x-3)(5-x)+(x-6)(x+7)-2(3x-11)+22=0$;

(ii) $x-\frac{1}{3}(x-2)=\frac{1}{4}(x+23)-\frac{1}{5}(10+x)$.

99. Solve

(i) $\frac{3x-3}{2}=\frac{3y-3}{4}$, $2x-\frac{y}{3}=7$;

(ii) $x+2y-3z=1$, $3x-y+z=2$, $5x+4y-7z=6$.

100. In a certain number of two digits, the digit in the units' place is less than the other by 3, and the number itself exceeds six times the sum of its digits by 7: find it.

101. Multiply $9x^2-3xy+y^2$ by $3x^2-2xy+y^2$.

102. Divide $3x^3+4ax^2-6a^2x-4a^3$ by $x+2a$.

103. Simplify
$$a^3-(a^2+7a)-\{a^3-2(3a+1)\}+\{a^3-(a+7)\}.$$

104. Find the G.C.M. of $2x^2+x-6$ and $8x^4-26x^3+21x^2$: find also the L.C.M. of the same two expressions.

105. Reduce $\frac{12x^4-3x^3-8x^2+2x}{3x^4-3x^3-8x^2+2x+4}$ to its lowest terms.

106. Extract to five decimal places the square root of 30.

107. Solve the equations:

(i) $12x+6(7-8x)=5(9x+3)-783$;

(ii) $\frac{x+9}{4}-\frac{3x-6}{5}=3-\frac{2x}{7}$;

(iii) $14x+9y=156,\quad 7x+2y=58.$

108. Solve

$5x-2y-3z=1,\ x-6y+4z=9,\ 7x+y-5z=14.$

109. A contribution of £125 was raised by A, B, and C: A gave twice as much as B, and B gave £15 more than C: what did each give?

110. Solve $x^2+16x=105$.

111. If $a=9$, $b=7$, and $c=1$, find the difference between $ab-b^2+ac-1$ and $3(bc+ac-7)$.

112. Subtract

$2x^3-8x^2y+13xy^2-y^3$ from $3x^3-8x^2y-xy^2+y^3$.

113. Multiply $3x^3-7x^2-1$ by $3x^2+7x+5$.

114. Divide $9a^5-31a^3-22a^2+19a+10$ by $3a^2+7a+5$.

115. Find the square root of

$a^6-4a^5+10a^4-20a^3+25a^2-24a+16$.

116. Solve the equation $3(x-4)+7x=18+4x$.

117. Solve the equation $\frac{7x}{3}+\frac{1}{2}=10\frac{5}{6}-\frac{x}{4}$.

118. Reduce $\frac{6x^3+3x^2-2x-1}{12x^4-9x^3-4x^2+3x}$ to its lowest terms.

119. Extract the fourth root of ·0016.

120. Solve $x^2+15x+26=0$.

121. Add together

$5a+7b-3c+d,\ 2c-5b-11a+8d,\ 3c-2b-10d+7a$.

122. Subtract $5x^2-3xy+2y^2-11yz+xz$ from
$6x^2+3y^2-4z^2-12yz+xz$.

123. Multiply $a^2+b^2-c^2$ by $a^2-b^2+c^2$.

124. Divide $x^4-12x^3+50x^2-84x+45$ by x^2-6x+9.

125. Solve the equation:
$$\frac{x+1}{2}+\frac{x+3}{4}=16-\frac{1}{3}(x+2).$$

126. Solve
$$3x+5y-6=0,\quad 2x-z+3=0,\quad 3y+2z-14=0.$$

127. Solve $2x^2+7x=39$.

128. Find the L.C.M. of
$2x^2+x-3$, $2x^2-9x+7$, and $3x^2-4x+1$.

129. Simplify $5x-3[2x+9y-2\{3x-4(y-x)\}]$.

130. Reduce $\dfrac{4x^3-17x+13}{6x^3-17x+11}$ to lowest terms.

131. Subtract the sum of
$5a^3-3a+2$, $3a-5a^2+7$, $3+4a^2-4a^3$, from $5-2a^3-a+2a^3$.

132. Multiply $a^5-3a^3x^2-6x^5$ by $2a^3+3a^2x-4x^3$.

133. Divide $243a^5-b^{10}$ by $3a-b^2$.

134. Extract the square root of
$1+4x+8x^2+6x^3-4x^5+x^6$.

135. Extract the square root of $1\cdot36\dot{1}$.

136. Find the G.C.M. of
$4(x^3-4x^2+9x-10)$ and $6(x^3+2x^2-3x+20)$.

137. Find the square root of 635·191209.

138. Solve $\dfrac{7x}{3}+\dfrac{4y}{9}=15$, $\quad 4x-\dfrac{3y}{5}=1\frac{1}{2}$.

139. Solve
$$4x+y-3z=15,\quad 7x+3y-4z=35,\quad x-5y=-15.$$

140. Solve $5x^2-8x=21$.

141. If $a=3$, $b=2$, $c=1$, $d=0$, find the value of
$$3a^2c-2abc+5c^2d+5ab^2.$$

142. Divide $32x^5+243y^5$ by $2x+3y$.

143. Extract the square root of
$$16a^{12}-24a^{10}b+25a^8b^2-12a^6b^3+4a^4b^4.$$

144. Extract the square root of $\cdot 00537$.

145. Solve the equations:

(i) $3(x-3)-2(x-2)+x-1=x+3+2(x+2)+3(x+1)$;

(ii) $\dfrac{2x+7}{7}-\dfrac{9x-8}{11}=\dfrac{x-11}{2}$.

146. Solve:
$$\frac{2x+y}{4}+1=\frac{2+x}{5};\quad \frac{3-4x}{6}+3=\frac{5y-7}{2}.$$

147. Solve $x+2y=7$, $y+2z=2$, $3x+2y=z-1$.

148. Solve $x^2+25x=-100$.

149. Divide 52 into two parts, so that the first may exceed twice the second by 4.

150. Two competitors, A and B, start in a walking race: four of A's strides exceed three of B's by two feet, and A takes seven steps while B takes six: the race ends in a dead heat. Find the length of A's stride.

151. If $a=0$, $b=1$, $c=2$, $d=4$, $e=8$, find the numerical value of
$$\frac{ad+be}{c}-\frac{b^3+2c^3}{d}+\frac{ad^2+c^3e}{3d}.$$

152. Divide $x^3-y^3-z^3+2yz$ by $x-y+z$.

153. Find the cube of $2x-5y^3$.

154. Solve

$$\left.\begin{aligned}\frac{x}{3}+\frac{2x+3y}{6}&=8\\ \frac{7y-3x}{2}-y&=11\end{aligned}\right\}.$$

155. Solve

$3x+2y-z=20,\ 2x+3y+6z=50,\ x-y+6z=35.$

156. Find the L.C.M. of

$$6x^2+7x-3 \text{ and } 12x^2+16x-3.$$

157. Simplify $2(a-b)-\{a+[b-2(a-b)]\}$.

158. Divide 150 into two parts such that if one be divided by 23 and the other by 27, the sum of the two quotients may be 6.

159. Find a number of two digits whose sum is 13, such that if 11 be added to the number, the places of the digits will be reversed.

160. Find the difference between $x^2+y^2+z^2$ and the square of $x-y+z$.

161. Divide $a^3+8b^3-c^3+6abc$ by $a+2b-c$.

162. Find the square root of 2910436.

163. Find the fourth root of

$$x^4-4ax^3+6a^2x^2-4a^3x+a^4.$$

164. Simplify $a-[b-\{2a-3(b-a)+c\}]$.

165. Solve the equations:

(i) $\frac{1}{7}(2x-3)-\frac{2}{9}(x+6)=\frac{x}{3}-5;$

(ii) $\frac{x}{3}-\frac{2y}{5}=\frac{3x-1}{2}-\frac{4y-1}{3}-1=0.$

166. Solve

$$3x-2y=4y-5z=2x+3z=2.$$

167. Reduce $\dfrac{12x^2(x^2+6x+5)}{32x^3(x^2+4x-5)}$ to its lowest terms.

168. Find the L.C.M. of

$$x^4-2x^3-3x^2+6x \text{ and } a^2x^3-2a^2x^2-5a^2x+10a^2.$$

169. Solve $5x^2+4=12x$.

170. A bill of £3. 7*s*. was paid with florins and half-crowns; if shillings had been used instead of half-crowns the whole number of coins used would have been 56: find the number of half-crowns.

171. If $a=3$, $b=2$, $c=1$, $d=0$, find the value of

(i) $a^3-2a^2b+3ac^2-4bcd+c^3$;

(ii) $\dfrac{2a+b}{4c}-\dfrac{2b+2c}{a}+\dfrac{a+c}{b}$.

172. Add up

$$13x^2-14xy-y^2,\ 5xy-9x^2-y^2,\ 11y^2-6x^2-15xy,\ 10x^2+24xy-3y^2.$$

173. Find the G.C.M. of

$$6(x^3-10x^2+17x+12) \text{ and } 4x(2x^3-19x^2+27x+20).$$

174. Solve the equations:

(i) $2(x-3)-5(x-4)=x-6$;

(ii) $\dfrac{x}{2}-\dfrac{x-14}{3}=\dfrac{20-x}{2}$.

175. Extract the square root of

$$x^4-6x^3y+25x^2y^2-48xy^3+64y^4.$$

176. Find the time between 8 and 9 o'clock when the hour and minute hands of a clock are at right angles.

177. Extract the square root of 746841·64.

178. Divide x^4-81y^4 by $x-3y$.

179. Solve the following equations:

(i) $\frac{x+9}{4}-\frac{3x-6}{5}=3-\frac{2x}{7}$;

(ii) $\frac{2x-y}{7}+3x=2y-6$, $\frac{y+3}{5}+\frac{y-x}{6}=2x-8$.

180. Solve

$4x-5y+2z=6$, $2x+3y-z=20$, $7x-4y+3z=35$.

181. Divide $x^3-2xy+y^2-z^2$ by $x-y+z$.

182. Shew that $\{(a+b)^2+b^2\}-\{(a-b)^2+b^2\}=4ab$.

183. Prove that if $a=3$, $\frac{2}{a-1}-\frac{1}{a^2-1}$ is equal to $\frac{7}{8}$; and prove the same thing if $a=-\frac{5}{7}$.

184. Solve the equation:

$$\frac{x-4}{3}-\frac{2x-5}{4}=\frac{3x-11}{5}-3\tfrac{1}{4}.$$

185. Solve

$\frac{2x-5}{3}-\frac{3y-1}{5}=4-\frac{2y+9}{3}$; $y-3\frac{1}{2}+x=13\frac{1}{2}-y$.

186. Multiply

$a^3+4b^3+9c^3+2ab+3ac-6bc$ by $a-2b-3c$.

187. Divide $x^4-4xy^3+3y^4$ by $x^2-2xy+y^2$.

188. A farmer bought 12 cows and 20 sheep for £335; he afterwards bought 10 cows and 26 sheep for the same sum, paying £2 each more for the cows, and 15*s.* each more for the sheep. Find what each cow and sheep of the first set cost.

189. Find the L.C.M. of x^3-1 and x^3+x-2.

190. Solve $\frac{x^2}{2}-\frac{x}{4}=\frac{x^2}{3}+\frac{x}{6}-\frac{1}{4}$.

191. If $a=3$, what is the difference between a^4 and $4a$? What is the result of adding x^3 and $3x^3$ together? What of adding x^3 and $3x^2$?

192. Divide a^4+4b^4 by $a^2+2ab+2b^2$.

193. Find the square root of ·001817939044.

194. Simplify

$$15a-\{3a-2(11b+5c)+5a-3(2c-a-4b)\}.$$

195. Solve the equations:

(i) $5(5x-3)-(25x-53)+8(8x-11)=10(6x-4)+6$;

(ii) $\frac{2x-y}{3}=1$; $\frac{x}{4}-\frac{y}{3}=\frac{x}{6}$.

196. Solve

$$8x+4y-3z=6,\ x+3y-z=7,\ 4x-5y+4z=8.$$

197. Solve $\frac{x^2-10}{5}=\frac{4-x}{3}$.

198. Find the G.C.M. of

$$(3x^2-12)y \text{ and } 6y^2(x^3-2x^2+5x-10).$$

199. A has five times as much money as B; he paid B £1. 3*s.* and then had only twice as much: how much had A at first?

200. A man bought some tea of two sorts, 3 lbs. of the first costing as much as 4 lbs. of the second, while 9 lbs. of the first and 8 lbs. of the second together cost £2. 5*s.*: what was the price of each per lb.?

ANSWERS.

Ex. I.

1. 9.	2. 1.	3. 39.	4. 36.	5. −20.
6. 31.	7. 2.	8. −21.	9. 50.	10. 16
11. −16.	12. −3.	13. 42.	14. 98.	
15. −44.	16. 7.	17. 184.	18. 36.	
19. 54.	20. 120.	21. 104.	22. 128.	
23. −69.	24. 74.	25. 1.	26. 3.	
27. 4.	28. 8.	29. −5.	30. $-4\frac{5}{8}$.	
31. 55.	32. −42.	33. 234.	34. 175.	
35. 225.	36. 1016.	37. 58.	38. 113.	
39. 466.	40. 142.	41. −586.	42. 596.	
43. 361.	44. 115.	45. 235.	46. −209.	
47. −28.	48. 1300.	49. $37\frac{1}{4}$.	50. $5\frac{1}{8}$.	
51. $14\frac{1}{8}$.	52. $25\frac{1}{8}$.	53. $\frac{1}{10}$.	54. $\frac{5}{12}$.	
55. 270.	56. $36\frac{7}{10}$.			

Ex. II.

1. $16x-4y+10$.
3. $8a^2+19a$.
5. $-14x^3-8y^3+2z^3$.
7. $38a-6b+d+6e$.
9. $13x^3-5x+5$.
11. $13x^3-25x^2+8x+5$.
13. $20a^2+11b^2+2c^2-4bc+12ca+2ab$.
15. $-5a^2+16b^2+4c^2-bc-10ca$.
17. $13x^2+13y^2-3z^2+2yz+10xy$.
19. $-15a^3+6a^2b-11ab^2+15b^3$.
21. 0.
22. $a^3-2a^2b+5b^3+2b^2c-6c^3$.
23. $5+15x-x^2+x^3+7x^4$.
24. $-4x^4+5x^3y+12x^2y^2-xy^3$.
25. $5x^4$.
26. $3a^4+15a^3+11a^2+14a-13$.
27. $16x^3+12x^2y+xyz-2y^3-4y^2z$.
28. $2abc-ac^2$.
29. $2a^4-2a^2x^2+4ax^3+x^2-13$.
30. 0.

Ex. III.

1. $11a+2b+c+3$.
3. $7a+b+c+14d$.
5. $x^3+2x^2y+6xy^2$.
7. $12x^2+32b^2+6c^2-26d^2$.
9. $-a^2+12b^2+5c^2+21$.
11. $3a+15b+12c-20$.
13. $x^3-5x^2y+4xy^2+2y^3$.
15. $4a^2+15b^2+7c^2-2bc+6ab$.
17. $-7a^2+17b^2+16c^2-4ca+11ab$.
19. $x^3+11y^3-2z^3+4x^2y+7x^2z-4xyz$.

21. $5a^3+3a^2-9a+11$. 23. $2x^3+2x^2y-3xy^2-y^3$.

25. $-2x^5+13x^4y-7x^3y^2+3x^2y^3-2y^5$.

27. $4x^3-16x+64$. 29. $16x^4-8x^3y-4x^2y^2-2xy^3$.

31. $a^3-2a^2b-6ab^2+2b^3$.

33. $-13a^5-6a^4b+2a^3b^2+7ab^4-2b^5$.

35. $-3x^3$. 36. $6x^5-6x^4-2x^3-6$.

37. $-3a^2x-8x^3$. 38. $a^4-8a^3x-6a^2x^2+3ax^3+4x^4$.

39. $14x-7$. 40. $x^5+x^3y^2$.

Ex. IV.

1. $6x^5$. 2. $-8x^7$. 3. $-15a^9$. 4. $42a^7$.

5. $28a^7b^5$. 6. $-48x^5y^7$. 7. $-24x^5z^9$.

8. $24a^7b^8c^5$. 9. $15a^3b^3$. 10. $28x^3y^4z^4$.

11. $15a^3b^2$. 12. $-20a^2x^3$. 13. $-18x^6y^3z^3$.

14. $10a^4b^3c^3d$. 15. $-28a^3b^3x^4z$. 16. $15ab^2cx^3z^3$.

17. $2a^5b^7$. 18. $-2x^6y^3$. 19. $-x^5y^3$.

20. $3ax^2y^5z$. 21. $15x^5-25x^4+15x^3$.

22. $-56x^7+48x^6+72x^4$.

23. $30a^8-42a^6-48a^4+30a^3$.

24. $-7b^5-56b^4+49b^3-77b^2+35b$.

25. $18a^2b-30ab-12b$.

26. $24a^6x^3-21a^5x^3+18a^4x^4-15a^3x^5+21a^2x^3$.

27. $-16x^6y^{12}+14x^4y^8-8x^3y^6+6x^2y^4$.

28. $24a^7b^3-21a^6b^4+18a^5b^5-21a^5b^4c+27a^4b^5c+3a^2b^3c^5$.

29. $-45x^3y^4z^5+35x^2y^5z^5+20x^2y^3z^8$.

30. $28a^5b^3c^6-36a^5c^8+48a^3b^3c^8-28b^4c^9$.

31. $4a^2+8ab+3b^2$. 32. $12x^2+17xy+6y^2$.

33. $20y^2-38yz+14z^2$. 34. a^2-b^2. 35. x^4-y^4.

36. $6a^3+7a^2-10a+25$. 37. $15x^3+32x^2+7x-12$.

38. $4x^3-7xy^2+3y^3$. 39. $8x^3+a^3$.

40. $21a^3-29a^2+19a-6$. 41. $6x^3-29x^2+37x-5$.

42. x^3-a^3. 43. $4x^3-14bx^2+18b^2x-9b^3$.

44. $12x^4-29x^3+9x^2+25x-25$.

45. $6a^4+a^3-29a^2+27a-9$.

46. $15a^4+16a^3b-30a^2b^2-13ab^3+20b^4$.

47. $8y^4+6by^3-17b^2y^2-6b^3y+9b^4$. 48. x^5+a^5.

49. $9x^5-13x^4y+14x^3y^2-26x^2y^3+53xy^4-48y^5$.

50. a^3b^3-1. 51. $6x^4-13x^3+8x^2+7x-20$.

52. $4x^4-10x^3-19x^2-17x-12$.

53. $6a^4+6a^3-5a^2-13a-3$.

54. $14y^4-36y^3+y^2-9$. 55. y^4+16y^2+256.

56. b^8-2b^4+1. 57. $a^8-a^6-13a^4+11a^2-2$.

58. $x^{12}-8x^6+16$. 59. $6y^{12}+5y^9+3y^6-19y^3-15$.

60. $6a^4-17a^3b+17a^2b^2-6ab^3-6b^4$.

61. $8x^4-22ax^3-6a^2x^2+11a^3x-5a^4$.

62. $10x^4-7x^3y-36x^2y^2-10xy^3+8y^4$.

63. $6y^8-13b^2y^6+5b^4y^4+4b^6y^2-2b^8$.

64. $x^5+x^4y-xy^4-y^5$.

65. $a^5-2a^4b-16ab^4+32b^5$.

66. $4x^5-6x^4-6x^2+17x-12$.

67. $9a^6+6a^5-6a^4-4a^3+9a^2+3a-2$.

68. $6x^5-2x^4y-9x^2y^3-y^5$.

69. $2x^6-3ax^5-9a^2x^4+4a^5x+6a^6$.

70. $2x^6-5x^5-4x^4+10x^3+6x-15$.

71. $2x^7-3ax^6-2a^2x^5+3a^3x^4+2a^4x^3-3a^5x^2-4a^6x+6a^7$.

72. $x^5-6x^3+x^2+8x-4$.

73. $2x^7-3x^6-x^5-4x^4+6x^3+8x^2-9x-3$.

74. $a^8-3a^6b^2+a^5b^3+2a^4b^4-3a^3b^5+ab^7-b^8$.

75. $a^{10}-2a^6bc+a^4b^2c^2+a^2b^3c^3-b^5c^5$.

76. $a^3+b^3+c^3-3abc$.

77. $a^3-b^3-c^3-3abc$.

78. $x^4+2x^2y^2-8y^4-18y^3z^2-9z^4$.

79. $a^3-b^3+27+9ab$.

80. $8x^3+27y^2+z^3-18xyz$.

81. $a^2+2ab+b^2$.

82. $4x^2+12xy+9y^2$.

83. $9x^2-24ax+16a^2$.

84. $16y^2-40by+25b^2$.

85. $a^2+b^2+c^2+2ab+2ac+2bc$.

86. $x^4-6x^3+17x^2-24x+16$.

87. $a^3+3a^2b+3ab^2+b^3$.

88. $8x^3-36x^2y+54xy^2-27y^3$.

89. $x^4-4x^3y+6x^2y^2-4xy^3+y^4$.

90. $16a^4+160a^3b+600a^2b^2+1000ab^3+625b^4$.

Ex. V.

1. $4x^3$. 2. $3a^4$. 3. $-2y^3$. 4. $-3y^7$.
5. $4x^7$. 6. $2x^6$. 7. $3b^7$. 8. $-13a^3bc$.
9. a^2b^3. 10. $-3yz$. 11. $2x^3-3x+4$.
12. x^4-4x+5. 13. $-2y^8+3y^4-4y^2$.
14. $2a^2-3a+1$. 15. $6b^4-5b+4$.
16. $3a^4b^2-2a^2b-1$. 17. $x^3+13x^2y-12xy^2+10y^3$.
18. $-2x^5-6x^3y^2-10xy^4$. 19. $3a^4-2a^2b^2-4b^4$.
20. $3ac-2b^3c^3+a^3b^4$. 21. $x+10$.
22. $y+10$. 23. $x+11$. 24. $a-10$.
25. $a-4$. 26. $a-7$. 27. $x+10$.
28. $x-4$. 29. $x-3y$. 30. $3a-b$.
31. x^2-4x+5. 32. x^2-7x-8. 33. a^2-5a+6.
34. a^2+7a-5. 35. $2y^2-10y-6$.
36. $x^2-2xy+y^2$. 37. $x^2-4ax+4a^2$.
38. $3x^2-5xy-4y^2$. 39. x^3-5x^2+3x-4.
40. $2x^3+3x^2y-4xy^2+5y^3$. 41. $2x^2+5x+4$.
42. $3x^2-4x-3$. 43. $5x^3-2x+3$.
44. $2y^2-6y-4$. 45. $3a^3-5a-5$.
46. $x^2-3xy-y^2$. 47. $x^2-5ax-5a^2$.
48. $2a^2-4ab-5b^2$. 49. $y^2-2by+b^2$.
50. $x^2-2xy+4y^2$. 51. $2a^2+3ab+5b^2$.
52. $a^2+2ab+b^2$. 53. $2x^2-5ax-2a^2$.
54. $3x^2+2xy-3y^2$. 55. $2x^2-3xy-y^2$.

56. $2x^2-5xy-5y^2$. 57. $2a^2-5ab-3b^2$.

58. $3x^2+2ax+2a^2$. 59. x^3-5x^2-2x+1.

60. $x^3-2x^2y+3xy^2-y^3$. 61. $a-b$.

62. $2x+3y$. 63. a^2-ab+b^2. 64. a^2+ab+b^2.

65. $9x^2+6ax+4a^2$. 66. $8a^3+12a^2b+18ab^2+27b^3$.

67. $8a^3-12a^2b+18ab^2-27b^3$. 68. $x^4+2x^2y^2+4y^4$.

69. $x+4$. 70. $y-3$. 71. x^2+xy+y^2.

72. $a^2-4a+16$. 73. $a^4+a^2b^2+b^4$.

74. $x^3+x^2y+xy^2+y^3$. 75. a^3-8b^3.

76. $x^4+x^3y-xy^3-y^4$. 77. $a^4-2a^3+8a-16$.

78. x^2-3x-1. 79. x^3-2x+4.

80. $2x^2-6x+5$. 81. $3x^2-5x-3$.

82. $x^6-x^5y+x^3y^3-xy^5+y^6$.

83. $x^3-2x^2y+2xy^2-y^3$.

84. $x^4+x^3y+x^2y^2+xy^3+y^4$. 85. $a^2-2ay+y^2$.

86. $a+b+c$. 87. $a^2+b^2+c^2-ab-ac-bc$.

88. $a^2+4b^2+c^2+2ab-ac+2bc$. 89. $a-b+c$.

90. $a^2-2ab+b^2-c^2$.

Ex. VI.

1. $3a$. 2. $4a^2b$. 3. $6a^3b^2c$. 4. $9a^4b^3c^2d$.

5. $12xy^3z^2$. 6. $10x^6y^4z^8$. 7. $8ax^3y^5$.

8. $a^3b^4xy^5$. 9. $5a^2b^3x^5y^4z^7$. 10. $11ab^2c^3x^4y^5z^{10}$.

11. $x+4$. 12. x^2+10. 13. a^3+6.

14. $3a+2$. 15. $4a^3x+5a$. 16. $11x^4y^3+3x^3$.

17. $5x^2-7$. 18. $6a^3-5$. 19. $10a^4-3$.

20. $12ab^3-9$. 21. $7x^2z-3a^2$. 22. $5ab^3-2c^3$.

23. $7x^3-11yz^3$. 24. $2-7x$. 25. $1-3ax^3$.

26. $\dfrac{4-3x}{3-5x^3}$. 27. $\dfrac{1-9x^3}{3-5xy^2}$. 28. $\dfrac{5a-3xyz}{7c-2x^3}$.

29. $\dfrac{x-5x^5y^3}{4x-7x^3y^3}$. 30. $\dfrac{3a^3x^5+7x}{5ax-8x^3}$.

31. $1+2x+2x^3$. 32. x^2-2x-1. 33. x^2+x+1.

34. $4x^2+2x+1$. 35. x^2-3x+9.

36. $5a^3+3a-1$. 37. $3a^4-7a^2+2$.

38. $11+3xy-5x^2y^2$. 39. $4-3ab^2+2a^2b^4$.

40. $9a^4+a^2b-4b^2$. 41. $2x^2+6x-9$.

42. $8-12x^2-9x^4$. 43. x^2-7x-9.

44. x^3-3x-4. 45. x^3-2x^2-9.

46. x^6-9x^3+1. 47. $4-3b+c$.

48. $8-4x+4y^3$. 49. $1+x+x^2+x^3$.

50. $a^6+2a^4x-a^2x^2+x^3$. 51. $x^3+2x^2y-xy^2+2y^3$.

52. $8a^6-8a^4b+6a^2b^2-4b^3$.

53. $2-2xy^2+8x^2y^4-8x^3y^6$.

54. $x^8-x^6y+x^4y^2-x^2y^3+y^4$.

55. $a^4-2a^3b+3ab^3-b^4$. 56. $x^5-x^3y^2+2xy^3-3y^5$.

57. $a+b$. 58. $2x+3$. 59. $3x^2-y$.

60. $2a-b^2$. 61. $2-5a$. 62. $1+4x^2y$.

63. $a-b$. 64. $x-2$.

Ex. VII.

1.	25.	2.	16.	3.	27.	4.	21.
5.	29.	6.	35.	7.	47.	8.	51.
9.	72.	10.	93.	11.	59.	12.	73.
13.	62.	14.	113.	15.	122.	16.	307.
17.	173.	18.	216.	19.	513.	20.	619.
21.	704.	22.	812.	23.	971.	24.	2092.

25.	2307.	26.	5062.	27.	7003.
28.	210·6.	29.	78·04.	30.	702·4.
31.	1500·1.	32.	2·1075.	33.	18·225.
34.	42638.	35.	7541·6.	36.	7·062.
37.	609·3.	38.	25·203.	39.	14·7021.
40.	703001.	41.	·5.	42.	·09.
43.	·08944....	44.	·77459....	45.	·002.
46.	·00632....	47.	·28284...	48.	·70710....
49.	·31622....	50.	31·62277....		
51.	1·58110....	52.	18·03884....		
53.	2·21359....	54.	7·12046....		
55.	9·01387....	56.	$\cdot\dot{1}\dot{3}$.	57.	$2\cdot\dot{6}$.
58.	$\cdot00\dot{6}$.	59.	$\cdot\dot{4}\dot{6}$.	60.	$\cdot0\dot{5}\dot{3}$.

Ex. VIII.

1. $a-6b$. 2. $2x^2-21y^2$. 3. $5a+10b-9c$.

4. $-x^2+8xy-11y^2$. 5. $15x^3-7y^3-6z^3$.

6. $9a^2-3ab-9ac-c^2$. 7. 0. 8. $-6z$.

9. $-2c$. 10. $10x^2+7y^2$. 11. $2a+1$.

12. $14x-5y$. 13. $4x-4y$. 14. $9a-11b$.

15. $13x^2-4xy+3y^2$. 16. $3x^2+16x+2$.

17. $-7x^2+2xy+2x+4y^2+3$. 18. $9x^2-11x+10$.

19. $2a-b-12c$. 20. $-9b^2$. 21. $-x$.

22. $x+9$. 23. $5x^2+3x-13$.

24. $-11a-6b+5c$. 25. $x+4y-12$. 26. -7.

27. $3a-b+c$. 28. $9x^2+6x+11$.

29. $2a^2+3ab-3b^2-bc+5c^2$.

30. $6x^2+4xy-2y^2+3z^2$. 31. $-2a-3$.

32. $3a^2+5a+7$. 33. $5a+8$. 34. $-5x+2y-z$.

35. $3a-3b-c$. 36. x^2-9x-4.

37. $3a-5b-7c$. 38. $9x^2-4xy$.

39. $-7x^2-13ax+5a^2$. 40. $3a^3-14a^2+7a-3$.

41. $26x-27$. 42. $-3x^3-18x+4$.

43. $3(a-6b+11c)$. 44. $-3a^3+5a^2-14a+31$.

45. $-39x^2+3x+56$. 46. $14x^2-5x+21$.

47. $-2a^2-5a+15$. 48. $3x^2+7xy-7xz+9y^2$.

49. $3(a^2+b^2)$. 50. $a(6a^2-27a-10)$.

51. $a^2-2ab+ac+3b^2+5bc+4c^2$.

52. $3x^2-xy+11xz+3y^2-18yz-6z^2$.

53. $7x^2-2xy+24xz+7y^2+6yz$.

54. $8x^2-5xy+10y^2+yz$. 55. $a(3a-2)$.

56. $3(33x-187)$. 57. a^2+b^2. 58. 0.

59. $2a(5a^3+a^2-6a+11)$.

60. $-4x^3+5x^2y-4x^2z+3xy^2+3xyz-6y^3$.

Ex. IX.

1.	4.	2.	33.	3.	48.	4.	13.	5.	73.
6.	50.	7.	3.	8.	99.	9.	1.	10.	672.
11.	2.	12.	1.	13.	4.	14.	4.	15.	3.
16.	2.	17.	4.	18.	5.	19.	1.	20.	3.
21.	-16.	22.	-33.	23.	-3.	24.	-4.		
25.	-5.	26.	-28.	27.	-3.	28.	-4.		
29.	-1.	30.	-6.	31.	$4\frac{1}{2}$.	32.	$11\frac{1}{3}$.		
33.	$1\frac{1}{5}$.	34.	$\frac{3}{4}$.	35.	$\frac{1}{5}$.	36.	$2\frac{4}{7}$.		
37.	0.	38.	$-3\frac{1}{2}$.	39.	$-44\frac{1}{2}$.	40.	$-\frac{2}{3}$.		
41.	2.	42.	4.	43.	0.	44.	2.		
45.	1.	46.	$1\frac{1}{3}$.	47.	$\frac{7}{8}$.	48.	$1\frac{1}{3}$.		
49.	$4\frac{1}{2}$.	50.	$3\frac{1}{4}$.	51.	$\frac{5}{14}$.	52.	$2\frac{1}{3}$.		
53.	1.	54.	-5.	55.	$4\frac{4}{7}$.	56.	$-4\frac{1}{4}$.		
57.	$3\frac{1}{3}$.	58.	$11\frac{1}{3}$.	59.	$-1\frac{2}{7}$.	60.	$-3\frac{1}{2}$.		
61.	10.	62.	5.	63.	4.	64.	10.		
65.	1.	66.	$1\frac{1}{2}$.	67.	0.	68.	2.		

69. $6\frac{1}{2}$.	70. 1.	71. 12.	72. 24.
73. 50.	74. 96.	75. 20.	76. 24.
77. 8.	78. 21.	79. $-\frac{1}{2}$.	80. 0.
81. 2.	82. $12\frac{11}{31}$.	83. $4\frac{1}{24}$.	84. 7.
85. 3.	86. $2\frac{1}{2}$.	87. $12\frac{2}{15}$.	88. $\frac{24}{113}$.
89. 4.	90. $2\frac{1}{12}$.	91. 4.	92. 7.
93. 0.	94. 1.	95. -7.	96. $\frac{4}{5}$.
97. $-\frac{1}{2}$.	98. $-3\frac{1}{3}$.	99. $-4\frac{1}{2}$.	100. $-2\frac{1}{2}$.
101. 0.	102. 1.	103. 2.	104. -2.
105. -3.	106. $2\frac{1}{3}$.	107. $-1\frac{1}{3}$.	108. -2.
109. -3.	110. 2.	111. $-1\frac{1}{3}$.	112. 1.
113. 2.	114. 3.	115. -9.	116. -1.
117. -4.	118. $-\frac{2}{3}$.	119. $-7\frac{1}{2}$.	120. $-\frac{1}{5}$.

Ex. X.

1. 4.	2. 40.	3. 18*s*.	4. 250.
5. 120.	6. 100.	7. 231.	8. 40.
9. 12.	10. £21; £63.		11. 68.
12. 675, 225.		13. £57. 10*s*.; £42. 10*s*.	
14. £1. 9*s*.; £1. 1*s*.		15. 48; 32.	

16. £35; £45; £80. 17. £266. 13*s*. 4*d*.
18. £8000; £6000; £6000. 19. 140.
20. 240. 21. 7. 22. 15, 25.
23. 36, 24. 24. 9*s*. 12*s*. 25. 78, 3.
26. 102, 90. 27. 30. 28. 12.
29. 42 years. 30. 20. 31. 14 years.
32. £1. 13*s*. 33. 60; 15. 34. £5. 15*s*.
35. £6. 5*s*. 36. £160; £80; £60.
37. £175; £225; £360. 38. £20; £10; £22. 10*s*.
39. £57. 40. 105; 45. 41. 240.
42. 20. 43. 70; 50. 44. 12; 2.
45. 13; 21. 46. 84; 60. 47. 46; 24.
48. £35; £43; £70. 49. £17. 10*s*.; £31. 15*s*.; £21.
50. 96. 51. 40. 52. 15*s*.; 10*s*.
53. 60. 54. 12; 10; 6. 55. 1 hr. 20′.
56. £525; £975. 57. £25. 58. 10.
59. 9. 60. 974.

Ex. XI.

1. 2; 5. 2. 5; 7. 3. 12; 3.
4. 3; 2. 5. 10; 8. 6. 7; −2.
7. 4; 5. 8. 2; 7. 9. 11; −3.
10. 5; 4. 11. 7; 5. 12. 20; 25.
13. 7; 4. 14. 5; 6. 15. −1; 5.
16. −2; −13. 17. 10; −17. 18. 5; 0.

19.	0; −7.	20.	7; −5.	21.	3; −1.
22.	−7; 2.	23.	0; −2.	24.	−2; −7.
25.	1; −1.	26.	6; −1.	27.	10; 7.
28.	5; 0.	29.	11; 2.	30.	1; 9.
31.	5; 1.	32.	−2; 7.	33.	8; 0.
34.	−2; −3.			35.	16; 6.
36.	15; 20.	37.	7; 4.	38.	10; −2.
39.	8; 0.	40.	−10; 7.	41.	6; 12.
42.	5; −3.	43.	6; 7.	44.	−2; 0.
45.	−2; −8.	46.	11; −1.	47.	$1\frac{1}{4}$; 2.
48.	0; −5.	49.	1; 6.	50.	6; −2.
51.	−1; −1.			52.	12; −10.
53.	4; 0.			54.	12; −50.

Ex. XII.

1.	7; 12.	2.	5; 7.	3.	17; 4.
4.	£24; £12.	5.	$4\frac{1}{2}d.$; $3\frac{1}{2}d.$	6.	12; 4.
7.	£2000; £1600.			8.	20; 32.
9.	53; 28.	10.	40; 12.	11.	30; 8.
12.	20; 16.	13.	11; 24.	14.	14 miles.
15.	64.	16.	84.	17.	63.
18.	232.	19.	£2; £2. 10*s*.		
20.	30 ft.	21.	240; 160.	22.	£710.
23.	42; 30.	24.	9; 7.	25.	60 yds.
26.	9; $7\frac{1}{2}$.	27.	£19; £41.	28.	30; 9*d*.
29.	400; 12*s*.	30.	162.		

Ex. XIII.

1. $x+1$.
2. $x+1$.
3. $3x+4$.
4. $5x+2$.
5. $3xy+5$.
6. $a-1$.
7. $ab-1$.
8. $2x-3$.
9. $7x-5$.
10. $3xy-7$.
11. $3x+8$.
12. $2x+1$.
13. $11x-1$.
14. $12x+11$.
15. $2a+5$.
16. $4x-a$.
17. $5x+4$.
18. $2x^3-x-1$.
19. $2x^3+5x+1$.
20. x^2-2x+3.
21. x^2-3x-5.
22. $2x^3-3x-1$.
23. $2x^2-7$.
24. $3x+11$.
25. $7x-1$.
26. $2x-9$.
27. $3x-4$.
28. $3x-4$.
29. $11x-2$.
30. $2x-a$.
31. $3x-7$.
32. $12x-7$.
33. $5x+4$.
34. $3x-1$.
35. x^3-2x+3.
36. $(2x^3-7)x$.
37. $2(x^2+x-2)$.
38. $2x(x^3+3x+1)$.
39. $x(3x+7)$.
40. $x(x^3-3x-2)$.

Ex. XIV.

1. $\frac{3a^3}{5b^3}$.
2. $\frac{5b}{8a}$.
3. $\frac{6x^5}{7y^2z^3}$.
4. $\frac{20b^4}{23c^4}$.
5. $\frac{y^3}{2az^2}$.
6. $\frac{2a^2z^5}{y^7}$.
7. $\frac{9by^3}{10a^2x}$.
8. $\frac{x^3z^2}{12ab^2y^2}$.
9. $\frac{21qx}{2py}$.
10. $\frac{6bkl}{5a^2m}$.
11. $\frac{x-1}{x+1}$.
12. $\frac{x+2y}{x-2y}$.

13. $\frac{x-3y}{x+8y}$.
14. $\frac{2a-3b}{2a+3b}$.
15. $\frac{x-2}{4x-7}$.
16. $\frac{x-7}{x+7}$.
17. $\frac{5x+3}{5x-7}$.
18. $\frac{p-1}{p-3}$.
19. $\frac{5x^2+3}{5x^2-6}$.
20. $\frac{x-7}{x^2+3x-2}$.
21. $\frac{x-1}{x^2+4x+16}$.
22. $\frac{4x-1}{12x^2+4}$.
23. $\frac{x^2+4x+5}{x^2+5x+4}$.
24. $\frac{x^2-5x+5}{x^2-x+1}$.
25. $\frac{x-3}{2x^2-7x+7}$.
26. $\frac{x-2a}{x^2-ax+a^2}$.
27. $\frac{3(x+2)}{x^2+5}$.
28. $\frac{2x+1}{3x+2}$.
29. $\frac{3x+y}{7x-y}$.
30. $\frac{4x-1}{5x+1}$.
31. $\frac{4x+3}{3x^2-4x}$.
32. $\frac{x+2}{x^2-2x+4}$.
33. $\frac{x^2+3x+4}{2(x^2+2x+3)}$
34. $\frac{2x-3}{3x^2+2x-3}$.
35. $\frac{2(7x-3)}{3y(3x-7)}$.
36. $\frac{3x+4}{2x^2-x-1}$.
37. $\frac{x+1}{3x+2}$.
38. $\frac{x^2-2x+3}{x^2+2x-3}$.
39. $\frac{a(5x-6)}{x(12x+13)}$.
40. $\frac{3p+q}{q(4p^2+2pq-q^2)}$.
41. $\frac{x^3(3x+2)}{4x^2-3x+2}$.
42. $\frac{(3x^2+2x+1)x}{2x^2-2x-3}$.
43. $\frac{3x^2+2}{x(4x-1)}$.
44. $\frac{x^3+2x^2+2x+1}{x^4+x^3+x^2+x+1}$.
45. $\frac{2x^2-6xy+8y^2}{5x^2+xy-4y^2}$.

Ex. XV.

1. $(x-2y)(x-4y)^2$.
2. $(x+12)(x-7)(x+9)$.
3. $(3x-4)(6x-4)(x-1)$.
4. $(2x-5)(x-1)(3x-4)$.

5. $(2x+3)(x+5)(4x-1)$. 6. $(x-16y)(x-7y)(x-2y)$.
7. $(2x-5)(x+4)(3x^2-5x-2)$.
8. $(x^3-4)^2(x^3-7x+4)$.
9. $2(x-1)(x^2+x-7)(x^3-8x+15)$.
10. $(x-3)(x^3-2)(x-12)$.
11. $(5x-3)(2x+11)(2x^2+7x+7)$.
12. $(x+3)(x^2-4x+5)(x^2-2x+3)$.
13. $(x+4)(x-3)(x-1)^2$. 14. $(x-7)(x+4)(x^2+2x+3)$.
15. $(x-5)(x-2)(4x^2-5x-5)$.
16. $(x+7)(x-10)(x^3-7x+10)$.
17. $(3x-7)(x^3-2x+3)(2x^2+5x-3)$.
18. $(a-3)(a+1)(a^2+a+1)$.
19. $(x-1)(6x-5)(3x^2+x+1)$.
20. $2x(2x+3)(x-5)(3x-1)$.
21. $(5x^3-1)(4x^3+1)(5x^3+x+1)$.
22. $x(2x-1)(2x^3-2x-3)(3x^2+2x+1)$.
23. $(a^2-3a+8)(a^2-8a+3)(3a^2-8a+1)$.
24. $(x+1)(x+3)(x^4-x^3+6x^2-6x+6)$.
25. $(3x-5)(2x^3-1)(2x^2+x+1)$.
26. $6(x-2)(x^2+2x-1)(x+1)$.
27. $(x-1)(3x-2)(4x^3-1)$.
28. $6(a-2)(a-3)(a^3-3a^2+2a+4)$.
29. $(x+1)(x^3-2x+3)(x^3-3x+2)$.
30. $(a^2-4)(a^2-1)^2(a^3-a+2)$. 31. $(x+1)(x+5)(x+6)$.
32. $(x-3)(x+5)(x-2)(x^2+1)$.
33. $(x-2)(x+2)(x-5)(x^2+4)$.
34. $(3x+4)(2x-5)(3x-4)(3x^2+x-2)$.
35. $(2x-1)(x^2+3)(x^3-x-1)$.

Ex. XVI.

1. 1, 2, 5.	2. 3, 2, 6.	3. 4, 5, 1.
4. 2, −2, 5.	5. 0, 4, −3.	6. 0, 0, −1.
7. 2, 3, 4.	8. 3, 1, 2.	9. 2, 3, −1.
10. 1, 2, 3.	11. 2, 2, 2.	12. 10, 2, 3.
13. 2, 1, 4.	14. 3, 0, 5.	15. $\frac{1}{2}$, 2, $2\frac{1}{2}$.
16. $4\frac{1}{2}$, 11, 13.	17. 2, −3, 0.	18. 4, −1, −2.
19. 7, 2, 0.	20. 3, −2, 7.	21. −3, 2, 5.
22. 8, −4, 5.	23. 10, 6, 0.	24. 20, −8, −6.
25. 10, −1, 5.	26. 5, 3, 8.	27. −7, 6, 14.
28. 3, 0, 5.	29. 4, −1, 7.	30. 8, −1, −2.

Ex. XVII.

1. ±9.	2. ±12.	3. ±11.
4. ±3.	5. ±6.	6. ±5.
7. ±6.	8. ±1.	9. ±2.
10. 0.	11. 2; −8.	12. 3; −13.
13. 1; −15.	14. 4; −24.	15. 4; −8.
16. 10; −14.		17. 3; −5.
18. 7; −9.	19. 10; −58.	20. 20; −120.
21. 14; −2.	22. 9; −3.	23. 6; −4.
24. 2; 0.	25. 5; 1.	26. 6; 2.
27. 18; 2.	28. 81; 3.	29. −1; −95.
30. −32; −10.		31. 2; −9.

32. 2; −5. 33. 3; −8. 34. 5; −14.
35. 13; −2. 36. 25; −4. 37. 4; −5.
38. 12; −13. 39. 11; 2.
40. 50; 3. 41. 2; −4. 42. 1; −4.
43. 1; −5. 44. 4; −25. 45. 2; −1.
46. 2; −8. 47. 5; −15. 48. 2; −11.
49. 4; 3. 50. 11; −6. 51. 2; $-3\frac{1}{2}$.
52. 9; $-9\frac{2}{3}$. 53. 4; $-5\frac{2}{4}$. 54. 7; $-7\frac{2}{7}$.
55. 4; $-4\frac{5}{10}$. 56. 6; $-5\frac{1}{4}$.
57. 2; $-2\frac{1}{3}$. 58. 4; $-4\frac{2}{5}$. 59. 2; $-2\frac{7}{12}$.
60. 2; $-1\frac{1}{3}$. 61. 3; 2. 62. 4; −1.
63. 3; −10. 64. 3; −6. 65. −8; $7\frac{3}{8}$.
66. 1; −5. 67. 5; −4. 68. 2; $-\frac{1}{2}$.
69. $1\frac{1}{2}$; −1. 70. −1; −2. 71. 7; $-1\frac{1}{2}$.
72. 2; −3. 73. 2; $-4\frac{2}{3}$. 74. 6; $-4\frac{1}{2}$.
75. 3; $1\frac{1}{2}$. 76. 7; $-6\frac{3}{5}$. 77. $2\frac{2}{11}$; −1.
78. $1\frac{11}{14}$; $-\frac{1}{2}$. 79. $\frac{7}{37}$; $-\frac{1}{5}$.
80. $4\frac{1}{3}$; $-1\frac{1}{5}$.

Ex. XVIII.

1. 37. 2. $16x-7y-3z$. 3. $3a-8b+10c$.
4. $6a^6-7a^4b-8a^3b^3+16a^2b^3-10ab^4+3b^5$.
5. x^3-2x^2-1. 6. x^7+x^4+x. 7. x^3-2x^2+3x.
8. $-2a$. 9. $2\frac{1}{2}$. 10. 4.

11. 128. 12. $13(a^3+a^2+a-1)$.

13. $3a^3-2a^2b+5ab^2+b^3+9$.

14. $x^6+3x^5-5x^4-4x^3+18x^2-8x-8$.

15. $2a-b$. 16. $x^4-x^3y+xy^3-y^4$.

17. $2-x-3x^2+2x^3$. 18. (i) 1. (ii) 4.

19. 18; 10. 20. 7804. 21. 62; $-1\frac{1}{3}$.

22. $18a^3-3a^2b+ab^2-2b^3$.

23. $2x^3+x^2y-5xy^2+10y^3$.

24. $6a^5-25a^4x+17a^3x^2-10a^2x^3+28ax^4-4x^5$.

25. $2x$. 26. $a^4-2a^3+3a^2-2a+1$.

27. $10x^3-7x^2+4x-3$. 28. (i) 12. (ii) 5; 2.

29. 100. 30. 4; 0; 5. 31. 1.

32. $-x^2-y^2+yz+z^2$. 33. $a^3+2a^2b-4ab^2-2b^3$.

34. $6a^4-20a^3b+11a^2b^2$. 35. $3x^3+2x^2-4x-1$.

36. $a^4+b^4+c^4-2a^2b^2-2a^2c^2-2b^2c^2$; $a^2-b^2-c^2-2bc$.

37. $4-a+5a^3$. 38. (i) 4. (ii) -2; 1.

39. $6x^2+4y^2-11z^2$. 40. $\dfrac{3x+1}{6x^2-7}$.

41. (i) 168. (ii) 339. 42. $5x^3-2xy^2$.

43. $6b$. 44. $4x^4-9x^2y^2+30xy^3-25y^4$.

45. $2x^2-3x-4$. 46. $a^3+2a^2b+2ab^2+b^3$.

47. $3x^3-2ax^2+3a^2x$. 48. $2x^3-17x^2+16x-3$.

49. (i) 9. (ii) 5; 5. 50. £60, £45, £24.

51. 6. 52. $4a^2-6ac+3b^2$.

53. $4a^5-6a^4-13a+3$.

54. $1+2x+3x^2+4x^3+5x^4$.

55. $-7x^3-9x+22$. 56. 8. 57. 5; 2.

58. $3a^2-2ab+3b^2$; 562·8. 59. $2x-9$.

60. 30. 61. 0.

62. $x^4-3x^3y-9x^2y^2+23xy^3-12y^4$; a^5-1.

63. $4a^2-6ab-8b^2$. 64. $2ab+4b^2$.

65. (i) 8. (ii) 5; 2. 66. $\frac{x+4}{3x^2-2}$.

67. $(3x-5)(x+1)(x-1)(x-2)$. 68. 1; 2; 4.

69. 60; 24. 70. 10; 7.

71. $-9x^3+2ax^2-31a^2x+16a^3$. 72. x^2+xy+y^2.

73. $a^4+a^3-2a^2+3a-1$; $27a^3-6ab+b^3+8$.

74. $3a-2b$; $2x^2y^2+2xy+1$.

75. $x^3-2x^2y+2xy^2-y^3$. 76. 2; 7.

77. 1; 1; -1. 78. ·01414...

79. £312. 10*s.*; £187. 10*s.* 80. 22 ft. 6 in.

81. $101\frac{3}{4}$. 82. $3a^5-19a^4b-9a^3b^2+5a^2b^2$.

83. $x^6-15x^4y+75x^2y^2-125y^3$.

84. $2a^2-3a+1$. 85. $a^2+15ab-9b^2$.

86. $x-1$; $(x-1)(3x-2)(3x^2-x-1)$.

87. (i) 8. (ii) 2; 3. (iii) 20; 10; 5.

88. 2·2. 89. $\frac{x^2-2}{2x^2-x-4}$. 90. 28.

91. $7x^3+17x^2y$. 92. $2a^3b^3-4a^2b^2-5ab+5$.

93. $14a^4-34a^3b-5a^2b^2+ab^3-6b^4$. 94. $a^2+2ab+b^2$.

95. $32a^5+16a^4x^2+8a^3x^4+4a^2x^6+2ax^8+x^{10}$.

96. 6. 97. a^3+2a^2-a+2.

98. (i) $4\frac{1}{3}$. (ii) 5.

99. (i) $4\frac{3}{4}$; $7\frac{1}{2}$. (ii) $\frac{1}{4}$; $-4\frac{1}{2}$; $-3\frac{1}{4}$.

100. 85. 101. $27x^4-27x^3y+18x^2y^2-5xy^3+y^4$.

102. $3x^3-2ax-2a^3$. 103. a^3-a^2-2a-5.

104. $2x-3$; $x^2(2x-3)(x+2)(4x-7)$.

105. $\frac{(4x-1)x}{x^2-x-2}$. 106. 5·47722...

107. (i) 10. (ii) 7. (iii) 6; 8.

108. 7; 5; 8. 109. £70; £35; £20.

110. 5; −21. 111. 5. 112. $x^3-14xy^2+2y^3$.

113. $9x^5-34x^3-38x^2-7x-5$.

114. $3a^3-7a^2+a+2$. 115. a^3-2a^2+3a-4.

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